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An Introduction to the Description And Evaluation of Microwave Systems Using Terminal Invariant Parameters

U.S. DEPARTMENT OF COMMERCE

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Errata Sheet For

NBS Monograph 112

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by

Glenn F. Engen

Page 6 — Equation (16), last factor should read:

$$\left| \frac{\Gamma_\ell - \Gamma_g^*}{1 - \Gamma_\ell \Gamma_g} \right|^2$$

Page 11 — Equation (32)

Insert equality sign

Page 14 — Third paragraph, third line,

Delete asterisk on $N_{a\ell}$

Page 21 — Equation (A-30)

Insert equality sign after first line

Equation (A-35), last factor should read:

$e^{\gamma_a z}$

UNITED STATES DEPARTMENT OF COMMERCE
Maurice H. Stans, *Secretary*

National Bureau of Standards • Lewis M. Branscomb

An Introduction to the Description and Evaluation of Microwave Systems Using Terminal Invariant Parameters

Glenn F. Engen

Institute for Basic Standards
National Bureau of Standards
Boulder, Colorado 80302

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Preface

This monograph gives the technical background and basis for a new method of evaluating certain performance parameters which are common to all microwave systems. A unique feature of the new measurement concept is its elimination of the precision waveguide and connector requirements for an important class of measurement problems. This feature results from a reformulation in terms of "terminal invariant" parameters.

The text of this monograph was previously submitted to the faculty of the Graduate School of the University of Colorado under the title, "A New Concept in Microwave Measurement Techniques," in partial fulfillment of the requirements for the degree of Doctor of Philosophy, Department of Electrical Engineering, 1969.

Abstract

The description and evaluation of microwave systems is usually by means of microwave circuit analysis, which may be regarded as an extension of the practice at lower frequencies. In order to insure its validity, it is necessary to postulate that the different components, which comprise the microwave system, are interconnected via uniform and lossless waveguide, and which is usually (but not necessarily) restricted to single mode operation. As a consequence, precision (uniform) waveguide and connectors are usually considered necessary elements for an accurate experimental evaluation of a microwave system.

It is possible to avoid this requirement in an alternative formulation where the description is based upon net power, instead of the complex traveling wave amplitudes. In this reformulation the basic parameters include available power, maximum efficiency (or intrinsic attenuation), and several different "mismatch factors." The important feature of these parameters is their "terminal invariant" property, i.e., their invariance to an arbitrary shift in the terminal reference surface (in an assumed lossless region).

In this way the precision waveguide and connector requirement is avoided for an important class of measurement problems. In addition the physical model, upon which the description is based, is a simple one which provides improved insight into mismatch errors and corrections.

Key words: Attenuation; impedance; microwave power; precision connector; terminal invariant.

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An Introduction to the Description and Evaluation of Microwave Systems Using Terminal Invariant Parameters*

Glenn F. Engen**

1. Background

The microwave art is characterized by the fact that a typical dimension of the associated apparatus, if measured in wavelengths of the attendant electromagnetic radiation, is of the order of unity. Because of this, retardation can no longer be neglected, and the usual circuit concepts, which are extensively employed at lower frequencies, are no longer completely valid.

There is, however, a type of circuit or network theory [1] which retains its usefulness, and which finds widespread application in microwave problems. For example, it would be difficult, if not impossible, in the immediate context, to assign a meaningful "equivalent circuit" to a slab of resistive material. However, if this slab is mounted in a uniform section of waveguide, it now becomes possible to assign an equivalent circuit in such a way that if this two-port device is incorporated in a given system, the net result on the operation is predictable.

A *sufficient* condition for the validity of such a circuit representation is that the different components be interconnected by ideal (lossless and uniform) waveguide, and of such length that the interaction is only via single mode.[2]

Notwithstanding the formal similarity between microwave and low frequency circuit representations, the field of microwave measurements remains distinct for at least two reasons. First, the lower frequency concepts of voltage and current lose much of their practical significance in the microwave region and the measurement of power assumes a fundamental role. Second, the other measurement objectives are usually limited to a determination (many times only in part) of the equivalent circuit parameters. This includes measurements of both impedance and attenuation.

Because the uniform waveguide requirement is implicit in the usual microwave circuit representation, it is not surprising that a review of the existing art finds this requirement deeply enmeshed in current measurement practice. The "standing wave machine," for example, includes a moving probe which samples the field at different longitudinal positions in a *uniform* line. The reflectometer, on the other hand, measures reflection coefficient and, in particular, gives a null output when terminated by a matched (reflectionless) load. The matched load plays an important role in the adjustment or design of such devices, and the existing methods of recognizing matched loads include motion within a uniform piece of waveguide. Finally, the usual definition of attenuation is tied to the reduction in power which results when the two-port device is inserted in a *matched* (reflectionless) system. The realization of a "matched" system, in turn, calls for the already cited impedance measurement techniques.

In brief, the field of microwave measurements represents a highly developed art, and the waveguide uniformity requirement, upon which microwave circuit theory is based, is strongly reflected in the existing measurement practice. Indeed, it is difficult to imagine how it could be otherwise . . . at least while attention continues to focus on the usual microwave circuit theory.

Another feature of the existing art which warrants consideration is the usual description of a microwave system which often starts (sometimes implicitly) with a model in which a match (reflectionless) is assumed for the different components. Then, in order to extend its validity, *mismatch* corrections are introduced, and here the circuit concepts come into full play.

Unfortunately, however, the mathematical descriptions which usually emerge from such an approach are often complicated in appearance, and provide little physical insight into the phenomena they describe. As a result, the subject of mismatch errors and corrections is but little appreciated, even by many practicing engineers. In addition, there is a considerable amount of permissible arbitrariness in interpreting the mismatch correction. This is reflected by the proliferation of terminology which exists. In a mismatched system, for example, what is the most meaningful way to characterize the generator power? Should it be (a) the power which would be delivered to a reflectionless load, (b) the power actually delivered to the given load, (c) the power associated with the emergent wave amplitude when the generator is terminated by the given load, or (d) the available power (which is obtained under conditions of conjugate impedance match)? All of these concepts (and perhaps more) are in current

*Based on the thesis "A New Concept in Microwave Measurement Techniques" in partial fulfillment of the requirements for the degree of Ph. D., University of Colorado, Boulder, Colorado. 1969.

**Radio Standards Engineering Division, Microwave Circuit Standards, NBS Laboratories, Boulder, Colorado 80302.

[1] See for example reference [1].

[2] The extension to multimode interaction is straightforward, however this type of operation is avoided in much of the current practice.

use. The term "matched load" has at least three meanings: (a) reflectionless, (b) equal to another load, and (c) complex conjugate of another load (such that maximum power transfer is effected). (In this dissertation the term "matched" is limited to the first of these.) This is only the beginning, however; a recent survey (which was by no means complete) indicates the existence of at least *seventeen* different terms for expressing the loss characteristics of a two-port device.

It must be recognized that this proliferation has been generated, in part, by the need for precision on the one hand, and on the other the desire to limit the number of terms required to describe a particular system. This has been achieved, but at the virtual expense of inventing a different language for each system.

In summary, the existing measurements art is characterized by a circuit theory which imposes (or at least does not relax) certain uniformity requirements upon the associated waveguide. The corollary precision connector requirement results from the recurrent need to separate this uniform waveguide into two (or more) parts. Perhaps the greatest application for this circuit theory is in connection with mismatch corrections or other attempts to sharpen the existing descriptions, but unfortunately there is no consensus of opinion on how best to do this, and a wide variety of competing descriptions are in current use. Finally, the inevitable deviations from the assumed uniformity, in any physically realizable apparatus, represents a limitation on the accuracy with which the performance or other characteristics may be evaluated.

2. Introduction

The "new" concept, which is to be developed in this dissertation, begins by observing that many of the microwave measurement objectives are, of themselves, quite independent of the assumed waveguide uniformity. Perhaps the best example is power which carries the same physical meaning in a system confined within irregular boundaries as in regular waveguide. In other cases, as will be developed, it is possible to substitute new measurement objectives, for old ones, such that the uniformity concept is avoided and yet retain a meaningful system evaluation. Traditionally, attenuation and impedance measurements have played a major support role in extending the dynamic range and evaluating mismatch corrections. However, since these quantities are tied to the uniformity concept, they will be replaced by others for which uniformity is not required. In particular this implies that the *matched load*, which plays a central role in much of the existing practice, is no longer required.

As a consequence, a more meaningful set of criteria, as to which are the important parameters in a mismatched system, will emerge, together with a new a model, for describing the system, in which the emphasis is on power transfer rather than traveling waves, and which provides improved insight into the mismatch corrections. Finally, the uniform waveguide and precision connector requirement will be greatly relaxed for the class of problems under consideration.

Throughout the development of this concept, it will prove convenient to retain the existing circuit descriptions as a working tool, and then examine the final techniques for their independence from uniformity considerations. The development will proceed with the consideration of specific measurement areas.

3. Microwave Power Measurement

Because power represents a basic parameter in the microwave art, it is appropriate to apply the new concept first to power measurement problems.

A large percentage of the microwave power meters in current use, especially at low power levels, are of the "terminating type." This means that they (ideally) terminate the waveguide or transmission line in its characteristic impedance and indicate the power which they absorb. The practical application of such devices calls for their connection to the signal source, in place of the load, to which the power input is required. Provided that the power meter and load are of identical impedance, the power meter reading will correctly indicate the power delivered to the load.

In the more general case, where the load and power meter are of different impedances, it is necessary to multiply the power meter reading by an appropriate "mismatch" factor [2][3]:

$$P_{gl} = P_{gm} \frac{|1-\Gamma_g\Gamma_m|^2(1-|\Gamma_l|^2)}{|1-\Gamma_g\Gamma_l|^2(1-|\Gamma_m|^2)} \tag{1}$$

where P_{gl} and P_{gm} are the powers delivered to the load and power meter and Γ_g, Γ_l, Γ_m are the reflection coefficients of the generator, load and power meter. The coefficient of P_{gm} in eq (1) is a mismatch factor, whose determination calls for the measurement of three complex reflection coefficients and the indicated computation. Fortunately, in many measurement applications (such as the transfer of calibration between power meters) the parameters of the generator are at one's disposal, and a substantial simplification in the mismatch factor is effected by adjusting the generator impedance such that $\Gamma_g = 0$. In the more general application, this is not possible, and in many cases the mismatch factor represents an error because of the practical problems in its evaluation.

The application of the new concept to this measurement problem begins with the system shown in figure 1 where that portion to the left of the arbitrarily chosen (and not necessarily plane) terminal surface is, by definition, the generator; that to the right is the load. The entire system,

[3] Figures in brackets indicate the literature references at the end of this paper.

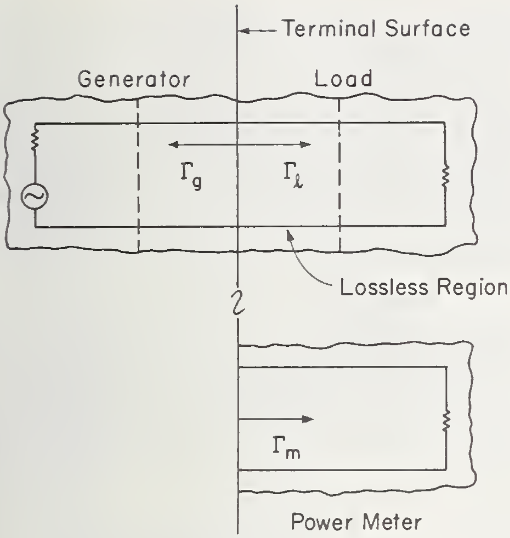

FIGURE 1. *Basic circuit involved in the application of a terminating type power meter.*

including both energy sources and sinks, is enclosed in a metal envelope,[4] a substantial portion of which, in practice, is in the form of waveguide. The object of the experiment is to replace the load by the power meter and from the power meter indication, predict the power delivered to the load. It is assumed that the input cross section of the power meter, while not necessarily the same as that for the load or generator, is such that the continuity of the metallic envelope is preserved, (i.e., the power meter mates with the generator in such a way that there are no "holes" or "gaps" in the system which would permit the electromagnetic energy to escape).

Of the several ways of characterizing the generator power, which were noted in a previous paragraph, only one satisfies the conditions of being (1) characteristic of the generator alone, (2) independent of the uniformity concept, and (3) independent of the choice of terminal surface (within the indicated lossless region). This is the *available power*, P_g, which is, by definition, the physically determined, maximum power that can be delivered to a passive load by the given generator.

In general, the conditions for maximum power transfer will not be satisfied, thus it is appropriate to write:

$$P_{gl} = P_g M_{gl} \tag{2}$$

where P_{gl} is the power delivered by the generator to the load and M_{gl} is a mismatch factor, whose value ranges between zero and unity, and whose physical interpretation is that of expressing to what extent the conditions for maximum power transfer have been satisfied. (Note that whereas P_g is a function only of the generator parameters, M_{gl} is a function of both generator and load. This is reflected in the choice of subscripts.) In terms of the usual microwave circuit theory, M_{gl} is given by:

$$M_{gl} = \frac{(1-|\Gamma_g|^2)(1-|\Gamma_l|^2)}{|1-\Gamma_g\Gamma_l|^2} \tag{3}$$

where the reflection coefficients Γ_g, Γ_l have been previously defined.

It is to be emphasized that while the uniformity concept and choice of terminal surface is implicit in the definition of the reflection coefficients, M_{gl} is independent of both,[5,6] and, in fact, retains its physical significance in applications where the usual definitions of reflection coefficient break down completely. In order to make these points more explicit, it is useful to consider the system shown in figure 2. As indicated, it is possible to consider the irregular region as part of the load, part of the generator, or divided between the two in an arbitrary way. Within the uniform portions of guide, existing theory permits the specification of P_g and M_{gl} in terms of circuit parameters. On the other hand, P_g and M_{gl} have been selected to characterize the system because they have a precise physical meaning which is independent of the choice of terminal surface.

The replacement of the load by the power meter is indicated by substitution of the subscript "m" for "l"

$$P_{gm} = P_g M_{gm}. \tag{4}$$

Provided that the power meter indicates the power which it actually absorbs,[7] its reading will equal P_{gm}, and if M_{gm} is known, P_g is determined by eq (4).

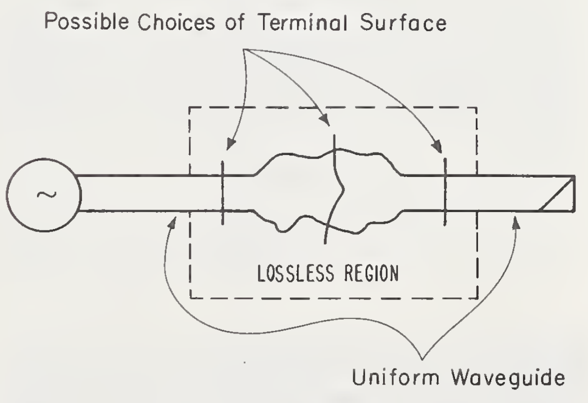

FIGURE 2. *A microwave system which includes nonuniform waveguide.*

[4] This is largely a matter of convenience for the discussion to follow. If the termination is an antenna it may in principle be replaced by an equivalent load whose characteristics are such that the relationship between the electric and magnetic fields, **E**, **H**, at the terminal surface is unchanged.

[5] The validity of this may be recognized in several ways. First, if the conditions for maximum power transfer are satisfied for one choice of terminal surface within the lossless region it must be satisfied for all choices, since if by any means it were possible to increase the power flow across one surface, energy conservation would require a similar increase at all surfaces within the lossless region. In a similar manner if $M_{gl} = 0.9$ in a given combination, this suggests that by a suitable deformation of the load (or generator) boundary (e.g., a waveguide tuning transformer), an additional 10 percent in power could be realized. This argument is evidently true for any choice of terminal surface within the lossless region.

Alternatively, the power P_{gl}, given by eq (2) cannot depend upon an arbitrary division of the system into generator and load. Thus, if P_g is invariant to the choice of terminal surface, the same must be true of M_{gl}.

[6] It is instructive to demonstrate this formally for a uniform system using conventional circuit theory. This is done in appendix 1.

[7] If, for example, the bolometric technique is used, the bolometer mount should be calibrated in terms of "effective efficiency" rather than "calibration factor."

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Finally:

$$P_{gl}=\frac{M_{gl}}{M_{gm}}P_{gm} \tag{5}$$

so that the mismatch factor introduced in eq (1) is the ratio of M_{gl} to M_{gm}. The evaluation of the mismatch correction in the use of a terminating type power meter has thus been reduced to the measurement of the two terms, M_{gl} and M_{gm}, which are independent of the uniformity concept. Thus, if a method can be devised for measuring M_{gl} directly, instead of relying upon the usual reflection coefficient measurements, it may be anticipated that such a technique might be independent of the waveguide uniformity requirement.

Before proceeding with a description of the technique for measuring these terms, it is desirable to briefly review the prior art.

3.1. Summary of Prior Art

For measurement purposes, one of the most useful of waveguide devices is the directional coupler. A pair connected as shown in figure 3,

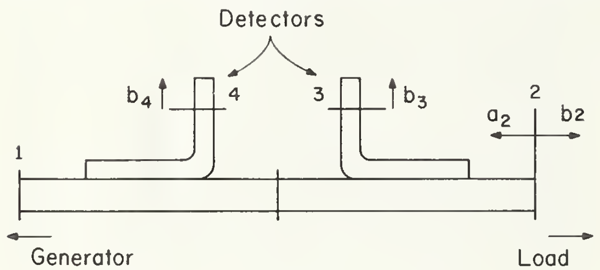


FIGURE 3. *Basic form of reflectometer.*

forms a reflectometer. The original use of the reflectometer was in conjunction with reflection coefficient measurement, and tuning procedures were developed [3] to compensate for imperfections in the coupler characteristics. More recently, the device has been used as a feed through [4] and as a variable impedance, terminating type [5] power meter.

In terms of the analysis employed in references [4] and [5], the electromagnetic fields, which obtain at the output port (2), are described by the complex amplitudes a_2, b_2 of the incident and emergent traveling waves respectively. These in turn are given by:

$$b_2=\frac{Ab_4-Cb_3}{\Delta} \tag{6}$$

$$a_2=-\frac{Bb_4-Db_3}{\Delta} \tag{7}$$

where, in a similar manner, b_3 and b_4 represent the amplitudes of the emergent waves at arms 3 and 4, and A, B, C, D, Δ are parameters of the four-arm junction and the detectors on arms 3 and 4. (Explicit equations for A . . . Δ in terms of the scattering coefficients are given in the cited references but will not be needed here.)

If the two couplers have perfect directivity, and are free of internal reflections, both B and C vanish and b_3 and b_4 become proportional, respectively, to a_2 and b_2. This is the required condition for a reflectometer. In general, the couplers do not satisfy these conditions, but the introduction of tuning transformers, T_x, T_y, shown in figure 4

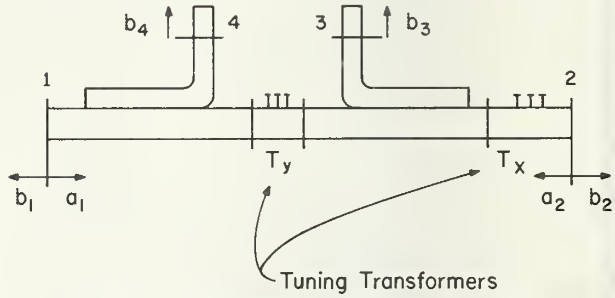


FIGURE 4. *Reflectometer with tuning transformers.*

makes it possible to adjust for the $B=C=0$ condition as described in reference [3].

For power measurement applications, the four-arm junction of figure 4 was fitted with bolometric power meters (rather than modulation detectors for example) on arms 3 and 4. With a suitable normalization, and assuming that $B=C=0$, the net (or actual) power P_2 emerging from arm 2 is given by

$$P_2=|b_2|^2-|a_2|^2 \tag{8}$$

$$P_2=\left|\frac{A}{\Delta}\right|^2|b_4|^2-\left|\frac{D}{\Delta}\right|^2|b_3|^2 \tag{9}$$

$$P_2=K_1P_4-K_2P_3 \tag{10}$$

where P_3 and P_4 are the powers measured at arms 3 and 4, and K_1, K_2 are constants of the system whose value may be determined by a suitable calibration or measurement procedure. Equations (8–10) may be given a simple interpretation: the (net) power emerging from arm 2 is given by the difference of the powers carried by the emergent and incident traveling waves.

Although the uniform waveguide concept was implicit in the derivation of this result, it was also recognized [4] that if the output port is extended by perfectly conducting boundaries of arbitrary geometry, and if the enclosed dielectric is lossless, the (net) power flow across *any* arbitrary terminal surface within this region is also given by eq (10). This conclusion follows, quite simply, from energy conservation. This result, in turn, implies that it should be possible to obtain eq (10) under a set of conditions which is less restrictive than $B=C=0$.

In particular, let the boundaries be extended by means of the (lossless) two-port device of figure 2. The resulting structure is shown in figure 5. If the terminal surface is shifted from the unprimed to

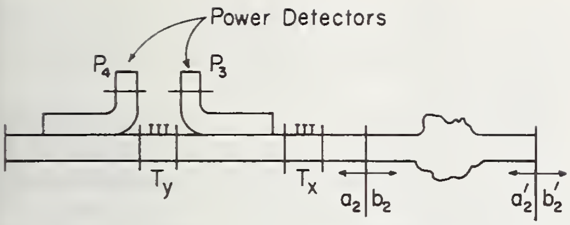


FIGURE 5. *Reflectometer with nonuniform output arm.*

primed position, the parameters $A \ldots \Delta$ will take on new values $(A' \ldots \Delta')$ and in general B' and C' will no longer vanish. It has been shown [4] however that if the structure satisfies the (uniformity) conditions under which $A \ldots \Delta$ may be defined, and if these parameters satisfy the equation

$$BD^* - AC^* = 0 \tag{11}$$

where (*) represents the complex conjugate, this is both necessary and sufficient for the output power to be expressible in the form of eq (10). In addition, this equation is also satisfied by the primed variables $A' \ldots D'$.

The usual method of achieving eq (11) is by means of tuner T_y, a moving short at port 2, and observation of the effect of the motion upon the ratio P_3/P_4. Provided that T_y has been adjusted such that this ratio remains constant, it can be shown [8] that eq (11) is satisfied. Although the moving short is usually provided with a uniform waveguide section, the only necessary conditions are that the device be free of loss and that the phase angle of the reflection be adjustable. Waveguide uniformity is obviously not required to achieve these conditions.

Having shown that the relationship of interest (eq 10), once established, continues to give the (net) power, irrespective of changes in terminal surface, even in a nonuniform region, it is thus significant that a method, which is also independent of the uniformity concept, exists for recognizing the proper adjustment of T_y which leads to eq (10).

In the usual reflectometer application, the only place, where a uniform section of waveguide is required, is at the output port (2).[9] Nominal deviations from this ideal, in other parts of the structure, may cause some change in the elements of the equivalent circuit, but these are accounted for in the tuning and calibration procedure. If, however, the device is to be used as a feed through power meter, the above arguments indicate, in addition, that the uniform section is not required in the neighborhood of the output port.[10] This provides the basis for transferring calibrations between terminating type power meters without regard for possible connector discontinuity [4]. The prior art thus includes examples which are in the full spirit of the new concept.

3.2. A Measurement Procedure for M_{gl} and M_{gm}

In common with the foregoing arguments, the measurement procedure will be developed in terms of the usual microwave circuit theory, and then examined for its dependence upon the uniformity concept.

If, as already noted, the junction parameters satisfy eq (11), the power output at port 2 may be expressed in the form of eq (10), where [4]:

$$K_1 = \frac{|A|^2 - |B|^2}{|\Delta|^2}, \tag{12}$$

$$K_2 = \frac{|D|^2 - |C|^2}{|\Delta|^2}. \tag{13}$$

In general, this condition may be realized by adjustment of the tuner T_y. This assumes that the couplers meet certain minimum conditions, which are well satisfied by available components, as to minimum directivity, etc.

It is next instructive to consider the equivalent generator which obtains at port 2 of the "reflectometer" in figure 5, if the power, P_4, measured by the detector on arm 4 is assumed to be constant or independent of the load on port 2, the actual generator parameters, etc. (In practice this may be achieved by means of a servo controlled attenuator between arm 1 and the actual generator.) This problem has been treated by the author in a prior paper [6]; the important result in the present context is that there exists an equivalent generator, at port 2, whose impedance is independent of the actual source of microwave energy.

Following the results obtained in the cited reference, it will prove useful temporarily to replace the actual generator on port 1 by an adjustable passive load, and excite the junction via arm 2. The passive load on arm 1 is first adjusted such that a null obtains at arm 4, and tuner T_x is next adjusted such that the input impedance at port 2 is equal to the impedance [11] of the generator of figure 1. (For the present, the various components are assumed to have uniform waveguide.) The condition imposed upon the "reflectometer," by this adjustment, may be obtained from eqs (6) and (7) by letting $b_4 = 0$, $b_2/a_2 = \Gamma_g$, whence:

$$C + D\Gamma_g = 0. \tag{14}$$

Let Γ_l represent the ratio a_2/b_2 which now obtains at port 2 with the generator reconnected at port 1. In general, the relationship between b_3/b_4, and Γ_l is

[8] See appendix 2.

[9] See appendix 3.

[10] Note however that single mode operation was implicit in the derivation of eq (10) and that whereas the output cross section at arm 2 may be such as to permit multimode operation, it is, in general, necessary to postulate the existence of a "mode filter" at some point in arm 2 in order to insure the validity of eq (10). In practice this may be achieved by including a length of guide whose cross section permits only single mode operation.

[11] To the extent that T_x may be considered lossless, this will not affect the adjustment of T_y which provides the $BD^* - AC^* = 0$ condition. In general, it is desirable to use an alternative procedure to be described in a following paragraph.

given by,

$$\frac{b_3}{b_4}=\frac{A\Gamma_l+B}{C\Gamma_l+D} \tag{15}$$

Substitution of eqs (11) and (14) and taking the absolute values gives:

$$\left|\frac{b_3}{b_4}\right|^2=\frac{P_3}{P_4}=\left|\frac{A}{D}\right|^2\cdot\left|\frac{\Gamma_l-{}_g{}^*}{1-\Gamma_l\Gamma_g}\right|^2 \tag{16}$$

The factor $|A/D|^2$ may be measured by substituting a totally reflecting termination for Γ_l, i.e., let $a_2/b_2=e^{i\theta}$ where θ is arbitrary. Then,

$$\left.\frac{P_3}{P_4}\right|_{\frac{a_2}{b_2}=e^{i\theta}}=\left|\frac{A}{D}\right|^2. \tag{17}$$

Now:

$$1-\frac{|\Gamma_l-\Gamma_g{}^*|^2}{|1-\Gamma_l\Gamma_g|^2}=\frac{(1-|\Gamma_g|^2)(1-|\Gamma_l|^2)}{|1-\Gamma_l\Gamma_g|^2}=M_{gl} \tag{18}$$

Equations (16), (17), and (18), thus provide an experimental method of measuring M_{gl}. The measurement of M_{gm} obviously requires the substitution of the power meter in place of the load.

A review of these equations shows that if $\Gamma_g=0$, the procedure leads to the usual reflectometer adjustment $B=C=0$ and thus to the value of $|\Gamma_l|^2$; in addition, if $\Gamma_g=0$, M_{gl} is given by $1-|\Gamma_l|^2$. The measurement procedure outlined above may thus be regarded as a generalization of the reflectometer technique, where instead of $|\Gamma_l|^2$, the measurement yields $|\Gamma_l-\Gamma_g{}^*|^2/|1-\Gamma_g\Gamma_l|^2$.

In order to clarify the terminology it appears desirable to reserve the term "reflectometer" for a junction which has been so adjusted as to measure reflection coefficients. The same general combination of couplers and tuners, but with a different adjustment (one example of which has just been described) will be called a "generalized reflectometer" or "*g*-reflectometer" for brevity.

This measurement procedure will next be examined for its dependence upon the uniformity concept.

3.3. Examination for Dependence Upon Uniformity

The measurement procedure outlined in the preceding paragraphs calls first for the adjustment of T_g and then T_x. However in order to avoid interaction between the two adjustments an alternate form of the *g*-reflectometer, in which the order of the couplers is reversed, is required. Before evaluating the dependence upon uniformity it is desirable to carefully outline the exact measurement procedure.

Referring to figure 6, the items to be considered include the original generator (1) and load (2) (for which P_g and M_{gl} are required), the *g*-reflectometer (3), an auxiliary directional coupler, tuner (T_a), and signal source (4), a sliding short (5), and power meter (6). These items are identified by the corresponding numbers in figure 6.

The first step in the procedure is the adjustment of T_x such that the generalized reflectometer will simulate the behavior of the actual generator (1). The desired adjustment may be recognized with the help of the auxiliary directional coupler, tuner (T_a), and signal source (4). In particular, the actual source of microwave energy in the generator (1) is turned off,[12] the generator (1) is then connected to the auxiliary directional coupler and signal source (4), and tuner T_a adjusted for a sidearm null. The generator (1) is next replaced by the *g*-reflectometer (3), the input port (arm 1) of which is terminated in an arbitrary passive load. The tuner T_g is next adjusted for a null in arm 4 ($P_4=0$) and tuner T_x is adjusted for a null in the sidearm of the auxiliary directional coupler. The *g*-reflectometer parameters now satisfy eq (14), and this completes the adjustment of tuner T_x.

A signal source is next connected to the input (arm 1) of the *g*-reflectometer and the output port (arm 2) is connected to the sliding short (5). Tuner T_g is then adjusted such that the ratio P_3/P_4 remains constant for all positions of the short. The *g*-reflectometer parameters now satisfy eq (11).[13] This completes the adjustment procedure.

The measurement consists of observing the ratio P_3/P_4 with the sliding short,[14] load (2), and power meter (6) connected in turn. This provides a measure of M_{gl} and M_{gm} as already described using eqs (16), (17), (18). Finally, the power meter (6) is connected to the generator (1), P_{gm} is measured, and P_{gl} calculated using eq (5). Note that the available power, P_g, may now be obtained from eq (3), while M_{gl} indicates what fraction of the available power is actually delivered to the load.

Returning to figure 6, it will now be assumed that the generator (1) and *g*-reflectometer (3) outputs *only* are of uniform rectangular waveguide. When connected to the auxiliary coupler (4) there may be a discontinuity (in guide cross section) at the mating surface; if so, the discontinuity is assumed to be identical for the generator (1) and *g*-reflectometer (3). Within the immediate vicinity of the mating terminal surface a complete description of the waveguide fields will usually require a number of modes, while at a greater distance the higher modes will have decayed (exponentially) to negligible amplitudes and the field may be completely described by the wave amplitudes of the lowest order forward and reverse traveling waves. Corresponding to the auxiliary coupler (4) sidearm

[12] In practice there is often substantial isolation between the actual source of microwave energy and the generator output terminal such that the generator "impedance" is essentially independent of the energy source. In any case, however, it is not within the scope of this dissertation to consider all of the practical questions which may be suggested by the prescribed measurement procedures.

[13] Although Tuner T_g was also used in obtaining the previous adjustment, it can be shown (see appendix 4) that the $C+D\Gamma_g=0$ condition is invariant to the adjustment of T_g.

[14] In actual practice it is usually desirable to observe P_3/P_4 with a high quality fixed short, rather than a sliding one.

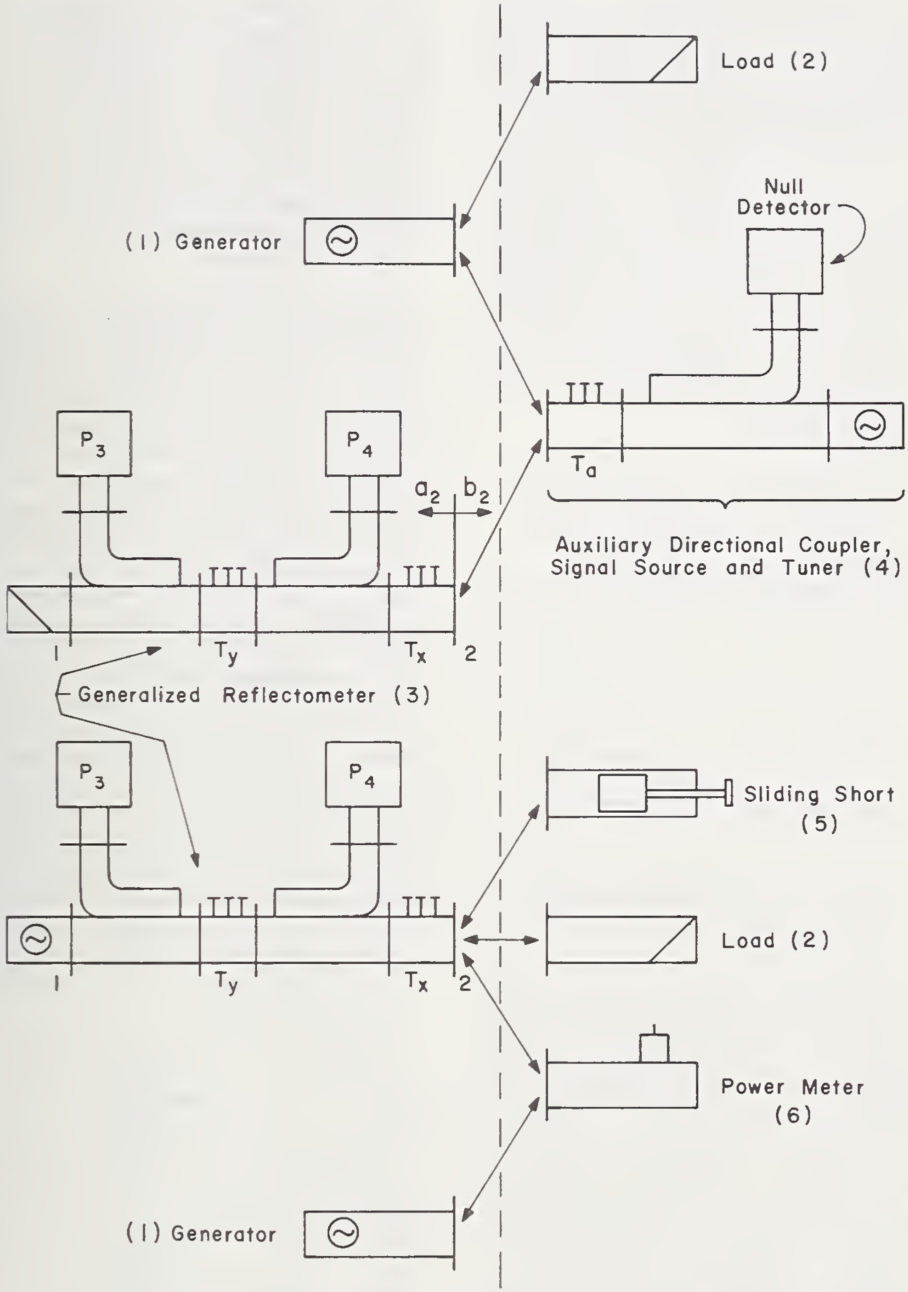


FIGURE 6. *Illustrating measurement procedure for* M_{g1}, M_{gm} *and* P_{g1}.

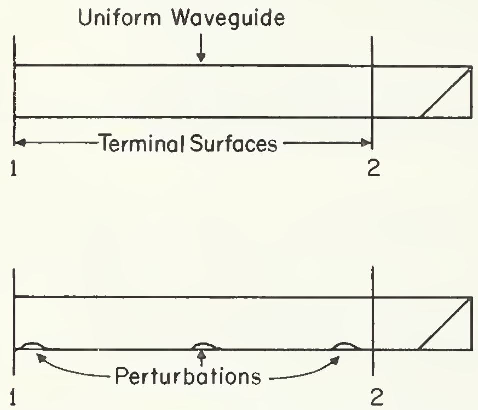
ull. there is a unique phase and amplitude relation-hip between these (lowest order) waves, and if his is the same for both generator (1) and *g*-reflec-ometer (3), their impedances (by definition) are qual. Thus, although uniform guide has been ostulated for the generator and *g*-reflectometer, is evidently not required in the auxiliary direc-onal coupler (4).

Continuing with this line of argument, it has ready been noted that uniformity is not required n principle at least) for the phasable short opera-on. Next, with regard to the load (2) and power eter (6). the fields in the mating uniform guide gain allow a modal expansion in which energy propagation is only via the lowest order mode. Associated with this mode is an impedance, and since the procedure is such as to explicitly account for this impedance, the operation is independent of deviations from the assumed uniformity and/or cross section of the load and power meter.

It is therefore a *sufficient* condition. for the prescribed measurement procedure, that the generator and *g*-reflectometer only be provided with the uniform waveguide section. In order to further relax this requirement, it is necessary to examine the role of the uniform section in greater detail.

Referring to figure 7, it will prove instructive to consider the operation of a uniform section of wave-

guide and passive termination. At the input port (terminal surface 1) it is possible to arbitrarily prescribe a transverse electric field $\mathbf{E}_t$ (subject only to the appropriate boundary conditions) and which, in principle, determines the electromagnetic fields throughout the entire structure. On the other hand, given an adequate separation between terminal surfaces[15] 1 and 2, and assuming all modes except one are beyond cutoff, the field at terminal 2 will (except for a constant multiplier) be substantially independent of the prescribed excitation at terminal 1 and dependent only upon the terminating load. Moreover, if the load possesses an adequate length of uniform waveguide at its input, the field at terminal 2 will consist essentially of the lowest mode traveling waves.



FIGURES 7 & 8. *Uniform and perturbed waveguide with termination.*

The validity of these statements may be easily recognized; if the arbitrary field at terminal 1 is expressed in terms of the appropriate modal expansion, it is found that the field at terminal 2 is due entirely to the single (lowest) propagating mode.[16]

The extension of these arguments to a nonuniform waveguide begins with a statement as to the types of structures which will be considered. Except as may be otherwise noted, it is assumed that the perturbed guide is at least a "reasonable" approximation to a uniform, single mode structure. In particular, this excludes arbitrary increases in cross section or the insertion of dielectric materials which would permit multimode propagation. In addition it precludes, for example, the use of a vertical septum in rectangular waveguide in such a way as to suppress the TE_{10} mode while leaving the TE_{20} unaffected. As a practical matter, this restriction is well satisfied by the components in common use. The standard tolerances on rectangular waveguide, for example, permit dimensional changes in the range 0.1 percent to 1.0 percent.

Now let perturbations be added to the uniform section of figure 7 as indicated in figure 8. Since the usual waveguide modes form a complete set, for any given cross section, one may visualize the arbitrary transverse electric field, which occurs at terminal 1 of the perturbed guide, in terms of its modal expansion using for basis functions those of the unperturbed waveguide. It is then instructive to consider the resultant field at terminal 2 on a mode-by-mode basis.

To a first approximation the perturbed guide will continue to propagate the dominant mode while suppressing the others. Thus, the field at terminal 2 will include traveling wave components associated with the dominant mode, and higher order, nonpropagating, components which are excited by the incident dominant wave, as required to satisfy the perturbed boundary conditions in the neighborhood of terminal 2. Higher order modal components which may be included in the prescribed excitation or generated in the vicinity of terminal 1 cannot make a direct contribution of significance to the field at terminal 2 because of the attenuation associated with their propagation. It may be possible to have mode conversion from the higher order modes to the dominant mode, if so this will combine with the already existing dominant mode to yield a change in the total field amplitude at terminal 2 . . . *but not in its functional form.* The physical description of the electromagnetic fields which emerges from these considerations, is that of the unperturbed, lowest order propagating mode, together with a superposition of higher order, nonpropagating modes, whose amplitudes at any particular cross section are determined primarily by the boundary conditions (perturbations) in the immediate vicinity.

In a similar manner, it is useful to consider the "impedance" which obtains at port 1 of the two guides. For the unperturbed case (fig. 7) the impedance is usually defined on a mode-by-mode basis in terms of the ratio of the amplitudes (reflection coefficients) for the ingoing and outcoming waves. For the lowest mode, this impedance will be a function of the load on terminal 2; for the higher modes, to the extent that the associated fields are attenuated before reaching terminal 2, the associated impedances are independent of the load.

This same formal definition of modal impedance can also be applied to the perturbed guide. Again it will be recognized that, for the lowest mode, the impedance is a function of the terminating load. In contrast with the uniform guide however, if the possibility of mode conversion is allowed, the impedances for the higher order modes will also be dependent upon the terminating load. Thus, if one termination is removed from the perturbed guide of figure 8, and another substituted in its place, this may (in principle) be observed as a change in impedance for several of the modes at terminal 1. How-

[15] Hereafter called terminals 1 and 2 for brevity.

[16] Loosely speaking, it is necessary to restrict the "arbitrary" field at terminal 1 to one in which the coefficients, of the higher order terms in the modal expansion, are of the same order or less than that of the dominant term. In the projected application this restriction is usually well satisfied.

ever if the characteristics of the second load are such that the lowest mode impedances observed at terminal 1 are identical, this is sufficient to ensure that the impedances (at terminal 1) are the same for each of the modes. The validity of this may be easily recognized from the given description where higher order mode interaction with the load is possible only through conversion to the lowest mode where, by hypothesis, the impedances are identical.

As already noted, the ratio of the ingoing to outcoming waves at terminal 1 is determined, on a modal basis, by the associated impedances. Thus, if the transverse electric field is given, the magnetic field is thereby determined.

For the purpose of this dissertation, two terminations are equivalent provided that for an arbitrarily prescribed transverse electric field at the input, the resultant transverse magnetic fields are the same. The foregoing arguments show that this is equivalent to requiring an equality of impedances on a mode-by-mode basis. Finally, for the type of structure under consideration (perturbed waveguide) it is possible to substitute one termination for another at port 2 and provided that the impedance observed at terminal 1 is unchanged for the lowest mode, the same will be true for all modes.[17]

The application of these results to the power measurement problem is straightforward. Referring again to figure 6, it has already been shown that a sufficient condition for the validity of the described measurement is that the generator (1) and *g*-reflectometer (3) outputs be provided with the uniform section. Consider next the arrangement of figure 9,

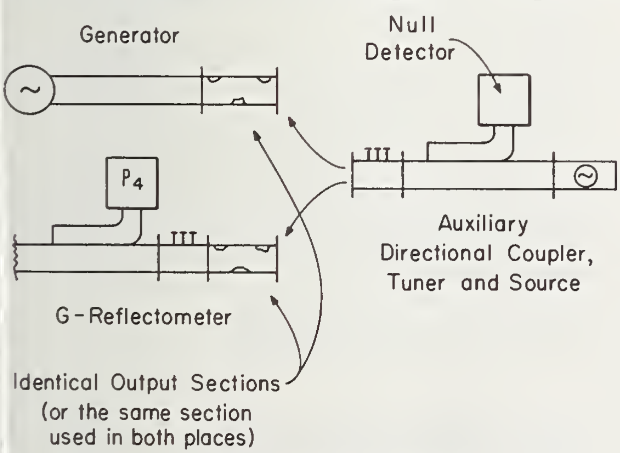


FIGURE 9. *Illustrating the extension of measurement capability to systems with nonuniform waveguide.*

[17] The foregoing arguments have been largely of a descriptive nature with the objective of providing physical insight into the problem. An analytical investigation of this topic is given in appendix 5.

[18] In practice it may be desirable to insure this condition hy actually removing a section of the generator output line and connecting it to the *g*-reflectometer.

[19] As thus defined, there is no place for "attenuation" in the new concept. In addition, however, the term "attenuation" is also associated in a general way with the loss characteristics of a two port device. Whenever this association is intended the term will be inclosed within quotation marks. Finally, in recognition of the prior terminology, the term "attenuator" will continue to be used to represent a two port device whose basic function is that of reducing the power delivered to the load.

[20] The maximum efficiency concept is also implicit in the terms intrinsic insertion loss [7], intrinsic attenuation [8], and possibly others.

where the generator and *g*-reflectometer are provided with identical output sections. Corresponding to the auxiliary coupler null a unique impedance will exist for the dominant mode and thus for all modes. Finally, if the same null obtains for the *g*-reflectometer, its impedance properties must be identical to the generator as required.

The requirement for a uniform waveguide section at the generator and *g*-reflectometer outputs is thus relaxed to the condition that they be of the same nominal length and cross section as the uniform guide and identical to each other.[18]

4. Microwave "Attenuation" Measurement

The application of these concepts to the area of attenuation measurement proceeds along the lines already established. As previously noted, at least 17 (and undoubtedly more) different terms have been introduced into the literature in order to describe the change (usually a reduction) in power which takes place when a two port device is inserted between a generator and load. In the most general case, the determination of this change in power level is by no means a trivial problem since, in addition to the three complex numbers (four if reciprocity is not satisfied) required to describe the two port (attenuator), the complex generator and load impedances are involved. A complete description of the problem thus involves 10 (or 12) parameters. Aside from a certain amount of duplication, the different terms, describing the power change which follows the insertion of the attenuator, have been generated by imposing different sets of boundary conditions on the system in which the attenuator is to be installed. "Attenuation," for example, is usually defined as the reduction in power which results if the associated system is perfectly matched.[19] This invokes the uniformity concept. "Insertion loss," on the other hand, is typically defined as the change in power (sometimes an increase) which results when the two port is inserted in an arbitrary system. This avoids the uniformity requirement but includes the parameters of the generator and load.

Among the descriptions which have been introduced, one satisfies the conditions of being characteristic of the two port alone, independent of the uniformity concept and choice of terminal surface. This is the maximum efficiency,[20] which will be denoted by η_a. (The letter η has been extensively employed to represent efficiency, the use of a single subscript reflects the fact that the term is a function of the parameters of the attenuator only and finally the choice of the letter "a" is a concession to the attenuation concept.)

In complete analogy to eq (2), and referring to figure 10, it is now convenient to write:

$$P_{gl} = P_g M_{ga} \eta_a N_{al} \tag{19}$$

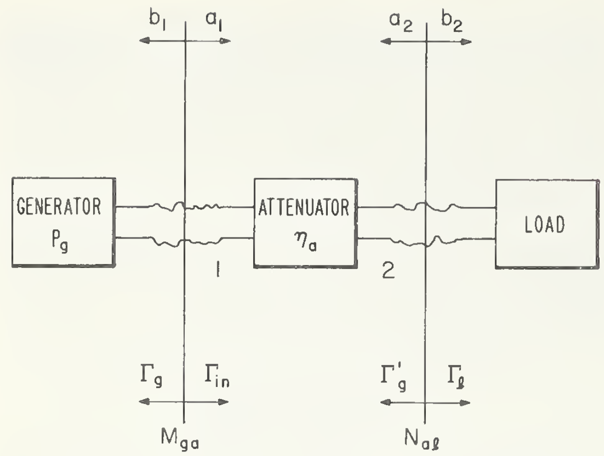


FIGURE 10. *Basic circuit for discussion of attenuation.*

where the terms are defined as follows:

P_{gl} = Power delivered by generator-through attenuator-to load, as previously defined.

P_g = Generator available power as previously defined.

M_{ga} = Mismatch factor between generator and attenuator-load combination (equivalent to M_{gl} except it is important to recognize that the impedance at the attenuator input is a function of both attenuator and load parameters).

η_a = Maximum efficiency of attenuator. (Except when $\eta_a=1$, it is obtained only for a uniquely determined value of load impedance.)

N_{al} = Ratio of the attenuator efficiency for the given load to its maximum efficiency. This is also a mismatch factor although of a different nature than M_{ga}.

In particular, (for a two-port) N_{al} is a function only of the attenuator and load parameters, whereas M_{ga} is a function also of the generator. This fact is reflected by the use of N rather than M to represent the mismatch factor. As in the previous case, each of these quantities is invariant to the choice of terminal surface within an arbitrary lossless region.

In terms of η_a and N_{al}, it is also convenient to write:

$$\eta_{al}=\eta_a N_{al} \tag{20}$$

where η_{al} is the efficiency of the attenuator of figure 10 when terminated by the given load. The application of the new concept to "attenuation" measurements will begin with a method for the measurement of η_{al}.

4.1. A Method for Measuring η_{al}

Referring again to figure 10 it is possible to express η_{al} in terms of the attenuator scattering coefficients and the load reflection coefficient Γ_l [21, 22]

$$\eta_{al}=\frac{|S_{21}|^2(1-|\Gamma_l|^2)}{|1-S_{22}\Gamma_l|^2-|(S_{12}S_{21}-S_{11}S_{22})\Gamma_l+S_{11}|^2}. \tag{21}$$

Although the scattering coefficients are extensively employed in microwave circuit analysis, an alternative description is more convenient in much of the discussion which follows. At terminal surface 1 the reflection coefficient, Γ_{in}, looking into the attenuator can be written

$$\Gamma_{\text{in}}=\frac{a\Gamma_l+b}{c\Gamma_l+1} \tag{22}$$

where: [23]

$$\left.\begin{aligned} a&=S_{12}S_{21}-S_{11}S_{22},\\ b&=S_{11},\\ \text{and}\quad c&=-S_{22}.\end{aligned}\right\} \tag{23}$$

The parameters a, b, c, thus provide a complete description of the attenuator if reciprocity ($S_{12}=S_{21}$) is satisfied.

It will also prove convenient to introduce the auxiliary quantity η_{rl} given by

$$\eta_{rl}=\frac{|a-bc|(1-|\Gamma_l|^2)}{|1+c\Gamma_l|^2-|a\Gamma_l+b|^2}. \tag{24}$$

Comparison of eqs (21), (23), and (24) indicates that η_{rl} and η_{al} are equal when reciprocity is satisfied.

The functional relationship between Γ_{in} and Γ_l, eq (22), is in the form of a linear fractional transformation, the geometry of which plays an important role in the discussion to follow. In general, let w and z represent two complex variables which are related by

$$w=\frac{\alpha z+\beta}{\gamma z+\delta}. \tag{25}$$

It can be shown [24] that a circle of unit radius and center at the origin in the z plane is mapped into a circle in the w plane whose radius R is given by

$$R=\left|\frac{\beta\gamma-\alpha\delta}{|\delta|^2-|\gamma|^2}\right| \tag{26}$$

and whose center is displaced from the origin by the (absolute) amount R_c,

$$R_c=\left|\frac{\beta\delta^*-\alpha\gamma^*}{|\delta|^2-|\gamma|^2}\right|. \tag{27}$$

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[21] See, for example, reference (9), p. 49.

[22] The term Γ_l introduced here is not to be confused with the Γ_l of the preceeding section.

[23] The a, b, introduced here are not to be confused with the a_1, b_1, a_2, b_2, which are used to represent the incident and emergent wave amplitudes at ports 1 and 2 etc.

[24] See a text on complex variable theory.

Now let port 2 of the attenuator be connected to the output port of a g-reflectometer as indicated in figure 11 and let the reflection coefficients looking to the right at terminal planes 1 and 2 of the attenuator be designated, by Γ_1 and Γ_2 respectively. (The nature of the load or other termination on port 1 which, produces Γ_1 is, as yet, unspecified.) In terms of these definitions, and noting that, as compared with figure 10, the direction of the attenuator has been reversed,

$$\Gamma_2 = \frac{a\Gamma_1 - c}{-b\Gamma_1 + 1}. \tag{28}$$

Substitution of this result into eq (15), and noting that the Γ_l of eq (15) now becomes Γ_2, yields:

$$\frac{b_3}{b_4} = \frac{(Aa - Bb)\Gamma_1 + B - Ac}{(Ca - Db)\Gamma_1 + D - Cc}. \tag{29}$$

The relationship between b_3/b_4 and Γ_1 is thus also in the form of a linear fractional transformation.[25]

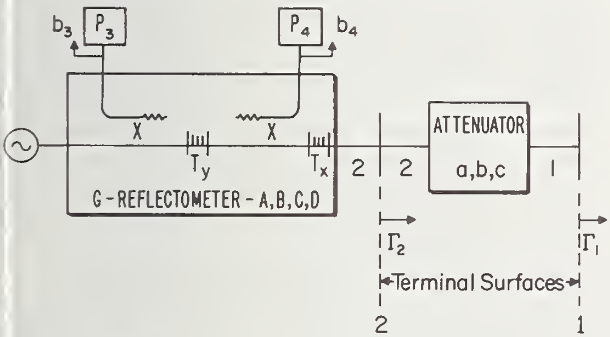

FIGURE 11. *Circuit for measuring* η_{a1} *and* η_a.

It is now instructive to consider the response of the g-reflectometer (of fig. 11) with arm 1 of the attenuator terminated by a sliding short. For a sliding short, $\Gamma_1 = e^{i\theta}$ where θ is a function[26] of the short position. The locus of Γ_1 is thus a circle of unit radius. Comparing eqs (25), (26), and (29), the transformed circle in the b_3/b_4 plane has a radius R_1 given by

$$R_1 = \frac{|(B - Ac)(Ca - Db) - (Aa - Bb)(D - Cc)|}{|D - Cc|^2 - |Ca - Db|^2} \tag{30}$$

while if the sliding short is connected to arm 2 of the g-reflectometer (in place of the attenuator) the corresponding radius R_2 is given by:

$$R_2 = \frac{|AD - BC|}{|D|^2 - |C|^2}. \tag{31}$$

Taking the ratio of eqs (30), and (31) yields, after some simplification,

$$\frac{R_1}{R_2} = \frac{|a - bc|\left(1 - \left|\frac{C}{D}\right|^2\right)}{\left|1 - c\frac{C}{D}\right|^2 - \left|a\frac{C}{D} - b\right|^2}. \tag{32}$$

Comparison of this result with eq (24) indicates that $R_1/R_2 = \eta_{al}$ provided that $C/D = -\Gamma_l$ and reciprocity holds. This type of boundary condition on C and D is identical to that introduced by eq (14) and may be effected by the tuning procedures for T_x already described and where the "load" of figure 10 takes the place of the (inactive) generator of figure 9. If the losses in the attenuator are small, the attenuator itself can take the place of the "identical output sections" of figure 9. Otherwise it is necessary to either remove part of the input lead (to the load of figure 10) and connect it to port 2 of the g-reflectometer or else assume the sections are identical. This aspect of the problem is completely analogous to that previously described in conjunction with figure 9.

Given the proper adjustment of T_x $(C = -D\Gamma_l)$ . . . note that an adjustment of T_y is not required, the efficiency η_{al} is thus given by the ratio of the radii of the two circles R_1 and R_2.[27] The measurement of these radii will be the subject of a later paragraph.

4.2. Measurement of η_a

The extension of these techniques to the measurement of maximum efficiency begins with a determination of that value of load impedance for which the efficiency is a maximum. This could be obtained from eq (21) by first choosing the argument of Γ_l (by inspection) such that the denominator is a minimum and then differentiating the resulting equation with respect to $|\Gamma_l|$. The same result, however, may be obtained by an alternative and somewhat shorter route. Returning to figure 10, if the parameters of the generator, attenuator, and load are taken as arbitrary, it would, in general, be possible to achieve an increase in the power delivered to the load by the introduction and appropriate adjustment of lossless tuners at terminals 1 and 2. The only exception would occur if the generator and load were such that a conjugate impedance match already existed at these terminals. Under these conditions, maximum power is delivered to the load, and the attenuator is evidently operating

[25] It is a general property that any succession of linear fractional transformations is expressible as a single linear fractional transformation.
[26] In the case of uniform waveguide, a linear function of the short position.
[27] The use of the impedance transformation properties, as a tool for the measurement of efficiency, has been described by a number of authors. (See references [10, 11, 12, 13].) However, the present method is useful with terminations of arbitrary impedance, avoids the measurement of impedance (in the usual sense), and, except for the reciprocity requirement, is applicable to two port devices of arbitrary characteristics.

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at maximum efficiency. Thus at terminal 1,

$$\Gamma_g{}^* = \Gamma_{in} = \frac{a\Gamma_l + b}{c\Gamma_l + 1} \tag{33}$$

while at terminal 2,

$$\Gamma_l{}^* = \frac{a\Gamma_g - c}{-b\Gamma_g + 1}. \tag{34}$$

Elimination of Γ_g between these equations yields

$$(c - ab^*)\Gamma_l^2 + (1 - |b|^2 - |a|^2 + |c|^2)\Gamma_l + (c - ab^*)^* = 0. \tag{35}$$

This represents the (implicit) condition on Γ_l for which the maximum efficiency is obtained. Equation 35 may be solved for Γ_l and the result substituted in eq (21) to yield an explicit expression for η_a. This result is not required in the present context, but is given in a following section.

Next, returning to the system of figure 11, it will be assumed that the parameters $A \ldots D$ satisfy eq (11) (as provided by the adjustment of T_y). In addition, let the ratio C/D $(= -\Gamma_l)$ be chosen in such a way that the circular locus, obtained with the sliding short at terminal 1, is centered at the origin of the b_3/b_4 plane. (Note that the circle obtained with the short on terminal 2 is centered by virtue of eq (11). The parameters of the g-reflectometer and attenuator combination may be obtained by inspection of eq (29). Substitution of these in eq (27), and assuming $R_c = 0$ yields:

$$|(B - Ac)(D - Cc)^* - (Aa - Bb)(Ca - Db)^*| = 0 \tag{36}$$

and imposing the condition of eq (11), and letting $\Gamma_l = -C/D$ yields:

$$|AD| \cdot |(c - ab^*)\Gamma_l^2 + (1 - |b|^2 - |a|^2 + |c|^2)\Gamma_l + (c - ab^*)^*| = 0 \tag{37}$$

Comparison of eqs (37) and (35) shows that the condition on the ratio C/D which provides $R_c = 0$ is identical to that for obtaining the maximum efficiency.

The procedure for measuring maximum efficiency, when reciprocity is satisfied, is thus as follows. Referring again to figure 11, port 2 of the g-reflectometer is terminated by the sliding short and tuner T_y adjusted such that the ratio $|b_3/b_4|^2 = P_3/P_4$ remains constant while the short position is varied. As previously noted, this establishes the $BD^* - AC^* = 0$ condition. The attenuator is next connected to the g-reflectometer (as shown in fig. 11) and the sliding short to the attenuator terminal 1. Tuner T_x is then adjusted such that P_3/P_4 again remains constant as the short position is varied. Provided that tuner T_x is lossless (as will be assumed for the present) the adjustment of T_y, as described, is invariant[28] to the adjustment of T_x. The g-reflectometer parameters are now such that eqs (11) and (37) are satisfied and the maximum efficiency is obtained in terms of the ratio of the radii of the two circular loci as already described for η_{al}.[29]

In practice, tuner T_x is not lossless and some interaction between the two adjustments is possible. It is noted, however, that the adjustment of T_y is required only as part of the procedure for obtaining the correct adjustment of T_x which, in turn, established the ratio of C/D corresponding to that value of Γ_l for which the maximum efficiency obtains. The efficiency measurement itself is independent of the adjustment of T_y. Thus, the only effect of losses in Tuner T_x is in obtaining an efficiency measurement for a load which differs from the one which yields the maximum efficiency. However, since the dependence of the efficiency upon the load impedance vanishes, as the maximum is approached, a first order error in the adjustment of T_x will yield only a second order error in the determination of η_a. This is expected to be completely negligible in practice.[30, 31]

In addition to reciprocity considerations, these techniques for measuring η_a and η_{al} are further restricted, by practical considerations, to rather small values of "attenuation" ~10 dB or less. This may be easily recognized from figure 11 where for large "attenuations" Γ_2, and thus the ratio b_3/b_4 (in terms of which the measurement is made), becomes independent of Γ_1. On the other hand, in their range of practical application, these techniques are easier to implement than the more general procedures which will follow. First, however, it will prove desirable to consider the measurement of N_{al}.

4.3. Measurement of N_{al}

As already noted, N_{al} is a mismatch factor, and is, by definition, equal to the ratio η_{al}/η_a. Thus the measurement techniques for η_a and η_{al}, already described, provide an implicit determination of N_{al}. However because of the practical limitations, already described, an alternative procedure is called for. This will now be developed.

As previously noted, N_{al} is a function of the attenuator parameters and of the terminating

[28] This may be recognized intuitively. If with the sliding short connected to arm 2, Tuner T_x is changed, this is equivalent (in terms of the system response) to motion of the short. However, if T_y is properly adjusted, the response is invariant to this motion.

[29] Alternative procedures for measuring η_a will be found in references (7) and (14).

[30] Indeed, this would probably be true, for many applications, even if the error were a first order effect.

[31] If for any reason a more accurate realization of the assumed tuning conditions (Equations (11) and (37)) is required, this could be achieved, despite the losses in T_x, by repeating the procedure several times. However, the need for this is not envisioned.

load (Γ_l). It is convenient, however to write [32]

$$N_{al}=\frac{(1-|\Gamma_l|^2)(1-|\Gamma_m|^2)}{|1-\Gamma_l\Gamma_m^*|^2-\eta_r^2|\Gamma_l-\Gamma_m|^2} \tag{38}$$

$$=1-\frac{(1-\eta_r^2)\dfrac{|\Gamma_l-\Gamma_m|^2}{|1-\Gamma_l\Gamma_m^*|^2}}{1-\eta_r^2\dfrac{|\Gamma_l-\Gamma_m|^2}{|1-\Gamma_l\Gamma_m^*|^2}} \tag{39}$$

where Γ_m is that value of load reflection coefficient for which the maximum efficiency is obtained, and η_r is given by eq (24) with Γ_m substituted for Γ_l. (If reciprocity is satisfied, $\eta_r=\eta_a$.)

Referring again to figure 11 it will be assumed that the parameters of the *g*-reflectometer satisfy the two conditions $C+D\Gamma_l=0$ and $BD^*-AC^*=0$. (The adjustment of T_x and T_y, by which this is obtained, is as previously described.)

In section 4.1 it was shown that η_{rl} is given by

$$\eta_{rl}=\frac{R_1}{R_2} \tag{40}$$

where R_1 and R_2 are the radii of the two circular loci obtained with the sliding short at terminal surfaces 1 and 2. It is also possible to express η_{rl} as follows: [33]

$$\frac{R_1}{R_2}=\eta_{rl}=\frac{\eta_r\left(1-\dfrac{|\Gamma_l-\Gamma_m|^2}{|1-\Gamma_l\Gamma_m^*|^2}\right)}{1-\eta_r^2\dfrac{|\Gamma_l-\Gamma_m|^2}{|1-\Gamma_l\Gamma_m^*|^2}}. \tag{41}$$

In addition to this expression, the determination of N_{al} also involves the ratio R_{c1}/R_2 where R_{c1} is the displacement, from the origin, of the circular locus obtained with the sliding short on terminal 1. In terms of η_r, Γ_l, and Γ_m, this ratio is given by: [34]

$$\frac{R_{c1}}{R_2}=\frac{(1-\eta_r^2)\dfrac{|\Gamma_l-\Gamma_m|}{|1-\Gamma_l\Gamma_m^*|}}{1-\eta_r^2\dfrac{|\Gamma_l-\Gamma_m|^2}{|1-\Gamma_l\Gamma_m^*|^2}}. \tag{42}$$

As a matter of convenience, in discussing eqs (41), (42), it will be convenient to make the following substitutions

$$\frac{R_1}{R_2}=u \tag{43}$$

$$\frac{R_{c1}}{R_2}=v \tag{44}$$

$$\frac{|\Gamma_l-\Gamma_m|}{|1-\Gamma_l\Gamma_m^*|}=g. \tag{45}$$

Equations (41), (42) now become:

$$u=\frac{\eta_r(1-g^2)}{1-g^2\eta_r^2} \tag{46}$$

$$v=\frac{g(1-\eta_r^2)}{1-g^2\eta_r^2} \tag{47}$$

Equations (46) and (47) may be solved for g and η_r in terms of the measurable quantities u and v. This is conveniently done by first squaring both sides of eq (47) and then eliminating g using eq (46). This leads to the following expression for η_r:

$$\eta_r=\frac{1}{2u}\{1+u^2-v^2-[(1-u^2-v^2)^2-4u^2v^2]^{1/2}\} \tag{48}$$

$$=u\left[1+\frac{v^2}{1-u^2-v^2}\left(1+\frac{u^2v^2}{1-u^2-v^2}+\frac{2u^4v^4}{(1-u^2-v^2)^2}+\cdots\right)\right], \tag{49}$$

while g is given by:

$$g=\frac{1}{2v}\{1+v^2-u^2-[(1-u^2-v^2)^2-4u^2v^2]^{1/2}\} \tag{50}$$

$$=v\left[1+\frac{u^2}{1-u^2-v^2}\left(1+\frac{u^2v^2}{1-u^2-v^2}+\frac{2u^4v^4}{(1-u^2-v^2)^2}+\cdots\right)\right]. \tag{51}$$

In terms of v and g, N_{al} is given by

$$N_{al}=1-vg \tag{52}$$

so that finally:

$$N_{al}=1-\tfrac{1}{2}\{1+v^2-u^2-[(1-u^2-v^2)^2-4u^2v^2]^{1/2}\} \tag{53}$$

$$=1-v^2\left[1+\frac{u^2}{1-u^2-v^2}\left(1+\frac{u^2v^2}{1-u^2-v^2}+\frac{2u^4v^4}{(1-u^2-v^2)^2}+\cdots\right)\right]. \tag{54}$$

[32] The proof of these equations will be found in appendix 6.
[33] This result may be obtained from eq (39), and using the relation $\eta_{rl}/\eta_r=N_{al}$.
[34] See appendix 7.

For a wide range of practical application the series converges rapidly[35] and eq (54) is the more convenient of the two expressions.

Equation (49) suggests an alternative method of measuring η_a when reciprocity is satisfied. However as previously noted, the accurate measurement of u becomes difficult when its value is small and this results in a first order error in η_a. On the other hand, as inspection of eq (54) shows, the value of N_{al} has a very small dependence upon u and the errors in its determination.

Comparison of eqs (38) and (39) with eq (18) indicates that for large values of "attenuation" (small values of η_a) the expressions for N_{al}^* and M_{gl} become identical if Γ_m is replaced by Γ_g. This is consistent with the relative independence of Γ_{in} upon Γ_l under the same conditions.

4.4. The Measurement of Small Values of η_a

As noted in a preceding section, the previously described techniques become impractical for small values of η_a. This will become more evident in the following section where the actual measurement procedures are explained in greater detail. This section will develop a technique which is appropriate for small values of η_a (large "attenuations") and which is also applicable when reciprocity is not satisfied.

Returning to figure 10, it will prove desirable to employ a description alternative to that associated with eq (19). In order to make this distinction explicit, however, the first description will be briefly reviewed. In eq (19) the product $\eta_a N_{al}$ is equal to the efficiency η_{al} which the attenuator provides when terminated by the given load. By definition, this efficiency is equal to the ratio of the power delivered to the load, to that delivered to the attenuator. The power delivered to the attenuator is, in turn, given by the product of the available power P_g and the mismatch factor M_{ga} between the generator and the attenuator-load combination. The analysis thus first associates the attenuator with the load (through the dependence of η_{al} upon Γ_l). The attenuator-load combination is then interpreted as a new "load" and the mismatch between it and the generator analyzed as was done for figure 1.

Notwithstanding the utility of this interpretation in many applications, an alternative analysis is more useful in the present problem. Instead of associating the attenuator with the load, its effect upon the system in figure 10 may be interpreted as modifying the generator parameters in such a way as to reduce the available power and, in general, of changing the source impedance. The mismatch M_{al}, between this new source (the generator-attenuator combination) and the load, may then be evaluated as was done for figure 1.

Let q_{ga} equal the ratio of the available power at terminal 2 (fig. 10) to that at terminal 1. It is then possible to write:

$$P_{gl} = P_g q_{ga} M_{al}. \tag{55}$$

In terms of the generator and attenuator parameters, q_{ga} is given by:[36]

$$q_{ga} = \frac{|S_{21}|^2(1-|\Gamma_g|^2)}{|1-S_{11}\Gamma_g|^2 - |(S_{12}S_{21}-S_{11}S_{22})\Gamma_g + S_{22}|^2}. \tag{56}$$

Comparison of this expression with that for η_{al} (eq (21)) indicates that the two are identical if Γ_g is substituted for Γ_l and the terms S_{11}, S_{22} are interchanged. Provided that reciprocity is satisfied, q_{ga} is thus equal to the efficiency which obtains for an assumed energy propagation in the reverse direction,[37] and with the generator impedance taking the role of a terminating load. The quantity q_{ga}, in its functional form, is thus closely related to efficiency, and this aspect will be exploited in the discussion to follow.

By definition, Γ_m is that value of Γ_l which satisfies eq (35). Solving this equation and substitution in eq (21) yields:

$$\eta_a = \frac{\dfrac{2|S_{21}|^2}{1+|S_{12}S_{21}-S_{11}S_{22}|^2-|S_{11}|^2-|S_{22}|^2}}{1+\left(1-\dfrac{4|S_{12}S_{21}|^2}{(1+|S_{12}S_{21}-S_{11}S_{22}|^2-|S_{11}|^2-|S_{22}|^2)^2}\right)^{1/2}} \tag{57}$$

In common with the foregoing treatment of efficiency, it will prove useful to write

$$q_{ga} = Q_{ga} q_a \tag{58}$$

where q_a is the maximum value of q_{ga} (obtained by varying Γ_g) and Q_{ga} is defined by eq (58). An explicit expression for q_a may be obtained, as was done for η_a, by first finding the value of Γ_g for which q_a is a maximum and then substituting in eq (56). If this is done it is found that,

$$q_a = \eta_a. \tag{59}$$

In a completely similar way it is found that the expression for Q_{ga} and N_{al} are identical, if Γ_g is substituted for Γ_l and Γ_m is understood to represent that value of generator reflection coefficient for which q_a is a maximum.

[35] For $|\Gamma_l|$ and $|\Gamma_{in}|$ both less than 0.2 (which corresponds to VSWR of 1.5) and of arbitrary phase, and with "attenuations" as small as 3 dB, the maximum error in N_{al} which results from truncating the series after the second term is less than 0.001 percent.

[36] At terminal 2, in figure 10, the equivalent generator reflection coefficient Γ_g' is given by $\Gamma_g' = (a\Gamma_g - c)/(-b\Gamma_g + 1)$ and the available power from the generator-attenuator combination is achieved when $\Gamma_l^* = \Gamma_g'$. Under these conditions $q_{ga} = M_{ga}\eta_{al}$. Combining these expressions and using eq (21) yields eq (56).

[37] In general the efficiency of a two-port is a function both of the terminating load and the direction of energy propagation. Except as otherwise noted, however, the assumed direction will be from left to right.

It is thus possible to drop the distinction between q_a and η_a or between Q and N and write eq (55) in the form:

$$P_{gl}=P_g N_{ga}\eta_a M_{al}. \quad (60)$$

The procedure for measuring N_{ga} is also identical to that for measuring N_{al} with the exception that it is now port 1 of the attenuator which is connected to the g-reflectometer. The technique for measuring small values of η_a, now to be described, is based upon eqs (2) and (60).

Referring to figure 12, the attenuator for which η_a is to be measured is connected between a g-reflectometer and a tunable power detector of wide dynamic range. It is assumed that tuner T_g of the g-reflectometer has been first adjusted such that eq (11) is satisfied, while the adjustment of T_x is abitrary. The tuning transformer which is part of the power detector is then adjusted for a maximum power indication,[38] so that the term M_{al} (eq (60)) is equal to unity.[39]

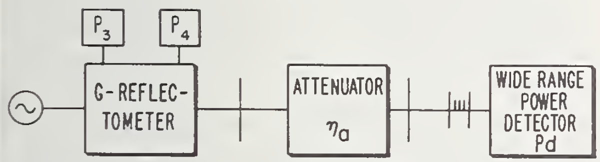

FIGURE 12. *Circuit for measuring small values of η_a.*

By hypothesis, the response of the power detector is proportional to the power which it absorbs. If this power indication is denoted by P_d, then

$$hP_d=P_{gl} \quad (61)$$

where h is a proportionality factor.

In terms of the notation employed in the section on power measurement, the available power, P_g, at the output of the g-reflectometer is equal to K_1P_4 where K_1 is, as yet, undetermined. After making these substitutions, eq (60) becomes:

$$hP_{da}=K_1P_{4a}N_{ga}\eta_a \quad (62)$$

where the subscript a has been added to P_d and P_4 to signify quantities measured with the attenuator inserted between the g-reflectometer and power detector as indicated in figure 12. In a similar manner, the subscript b will denote the observed values with the attenuator removed.

[38] More correctly, the adjustment is such that the power detector indication is a maximum relative to P_4 of the g-reflectometer.

[39] It is not necessary to assume that the transformer which provides this adjustment is lossless since this is taken care of in the procedure to follow.

Then,

$$hP_{db}=K_1P_{4b}M_{gd} \quad (63)$$

and taking the ratio of eqs (62) and (63) yields,

$$\eta_a=\frac{P_{da}P_{4b}M_{gd}}{P_{db}P_{4a}N_{ga}}. \quad (64)$$

The efficiency η_a is thus determined in terms of the observed quantities P_{4a}, P_{4b}, P_{da}, P_{db}, and in terms of M_{gd} and N_{ga}. These last two quantities are measured as previously described. In brief, the measurement of N_{ga} calls for the replacement of the power detector by a sliding short and certain measurements of P_3 and P_4, while the measurement of M_{gd} requires the values of P_3 and P_4 with the power detector and the short connected directly to the g-reflectometer.

It is of interest to compare this technique with the existing art. In the usual attenuation measurement, the g-reflectometer of figure 12 is replaced by a matched generator and the power detector also adjusted for an impedance match. The attenuation is then obtained from the ratio of the power detector readings, with and without the attenuator inserted. The wide dynamic range for the detector is usually achieved by means of another attenuator which may operate either at the signal frequency or, if hetrodyne detection is employed, at an intermediate frequency.

If one considers the g-reflectometer of figure 12 as the counterpart of the necessary instrumentation for establishing the impedance match in the attenuation measurement, the instrumentation requirements of the two measurements are nominally equivalent. The maximum efficiency measurement, however, requires a number of additional observations, most of which are associated with the determination of M_{gd} and N_{ga}.

4.5. Measurement of R and R_c

As outlined in the preceding sections, the measurement of η_{al}, η_a, N_{al} has been reduced to the determination of the parameters R and R_c (eqs (26) and (27)) for the linear fractional transformation, which relates the ratio b_3/b_4 to the reflection coefficients which obtain at the different terminal surfaces. It is the purpose of this section to describe the measurement procedure in greater detail.

It has been previously noted that the response of the ratio b_3/b_4 to the motion of the sliding short is in the form of a circular locus whose radius is R and whose center is displaced from the origin, in the b_3/b_4 plane, by the amount R_c. The relationship between these quantities is as indicated in figure 13.

In actual practice, arms 3 and 4 of the g-reflectometer are provided with power detectors such that the measured quantities P_3 and P_4 are proportional to $|b_3|^2$ and $|b_4|^2$ respectivelv. Since only

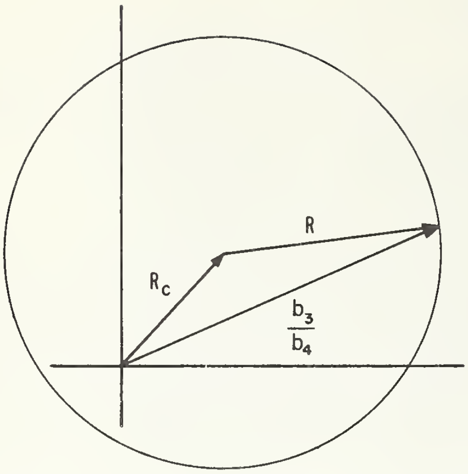

FIGURE 13. *Illustrating the relationship between* R, R_c, *and* b_3/b_4.

power ratios are involved, these factors may be taken as equal to unity (as was also done in eq (16)).

As the sliding short is moved, the ratio P_3/P_4 thus varies and inspection of figure 13 indicates that

$$\left|\frac{b_3}{b_4}\right|_{\max}=\left(\left.\frac{P_3}{P_4}\right|_{\max}\right)^{1/2}=R+R_c \tag{65}$$

$$\left|\frac{b_3}{b_4}\right|_{\min}=\left(\left.\frac{P_3}{P_4}\right|_{\min}\right)^{1/2}=\pm(R-R_c) \tag{66}$$

where the upper sign is used if, as shown in figure 13, $R \geqslant R_c$; otherwise the lower sign is used.

These equations may be solved to yield:

$$R=\frac{1}{2}\left[\left(\left.\frac{P_3}{P_4}\right|_{\max}\right)^{1/2}\pm\left(\left.\frac{P_3}{P_4}\right|_{\min}\right)^{1/2}\right] \tag{67}$$

$$R_c=\frac{1}{2}\left[\left(\left.\frac{P_3}{P_4}\right|_{\max}\right)^{1/2}\mp\left(\left.\frac{P_3}{P_4}\right|_{\min}\right)^{1/2}\right]. \tag{68}$$

Measurement of the maximum and minimum values of the ratio P_3/P_4 thus determines R and R_c but with an ambiguity as to which is which. As a practical matter, this ambiguity can often be resolved on the basis of an approximate knowledge of the magnitudes of the parameters involved. If not it can be easily resolved by introducing a variable attenuator ahead of the sliding short, and using the upper or lower signs depending respectively upon whether or not, by a suitable adjustment of the attenuator and short position, it is possible to make b_3 vanish.[40]

The earlier comments about the practical limitations in certain of the described techniques for measuring small values of η_a and η_{al} may now be better appreciated. Referring to eq (67), it is the lower sign which will usually apply if η_{al} is small, thus the quantity R, to which the efficiencies are proportional is the difference between two measurements, which individually may be rather large as compared with their difference.[41] A similar observation is true for the determination of R_c when it is small with respect to R. The inability to accurately measure small values of R_c is, however, of no concern since, as eq (54) indicates, $N_{al}\simeq 1-R_{c1}^2/R_2^2$ and thus the dependence upon R_c vanishes as R_c approaches zero.

4.6. Cascade Connected Attenuators

For completeness, it should be noted that there is a third type of mismatch factor which is of interest in conjunction with cascade connected attenuators.

Referring to figure 14 the power delivered to the load can be written:

$$P_{gl}=P_gM_{ga_1}\eta_{a_1}N_{a_1a_2l}\eta_{a_2}N_{a_2l} \tag{69}$$

where except for the addition of the subscripts 1 and 2, to identify the two attenuators, the terms have been previously defined. Note however that in the fourth term the additional subscript l is required to indicate that it is the mismatch between attenuator 1 and the attenuator 2 and load combination, which follows, which is meant. (In an alternative formulation, one might be interested in the mismatch between attenuator 2 and the generator-attenuator 1 combination which precedes it.)

Alternatively, the power delivered to the load in figure 14 may be written (cf eq (19))

$$P_{gl}=P_gM_{gA}\eta_AN_{Al} \tag{70}$$

where the two attenuators are now considered to comprise a single unit as reflected in the subscript A.

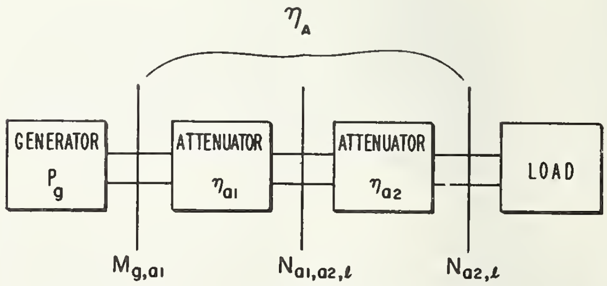

FIGURE 14. *Basic circuit for discussing cascade connected attenuators.*

[40] The basis for this test is as follows. In the usual microwave circuit description, a sliding short represents a reflection coefficient of unit magnitude and variable phase. In appendix 8 it is shown that reflection coefficients of magnitude less than unity are mapped into the *interior* of the circle of figure 13. Thus if the circle encloses the origin it will be possible to make b_3 vanish for a suitable reflection as provided by the attenuator-short combination.

Note that it is not necessary to carry this procedure to the point where the null is actually achieved. Starting from the point where the ratio $|b_3/b_4|$ is a minimum, the introduction of dissipative loss (via the attenuator) will increase the value of $|b_3/b_4|$ if the origin is outside the circle, decrease it, if the origin is inside.

[41] In the measurement of η_{al}, as prescribed in section 4.1, the adjustment of T_y was arbitrary, and this could in principle be used to make R_c vanish, such that the + sign would apply in eq (67). In the measurement of η_a the specified adjustment is, ideally, such that R_c vanishes. There is, however, a practical limitation as to the precision and stability with which the required adjustment can be made. This limits the techniques of sections 4.1 and 4.2 to small values of attenuation.

In terms of the individual efficiencies η_A may be written,

$$\eta_A = \eta_{a_1} R_{a_1 a_2} \eta_{a_2} \tag{71}$$

where $R_{a_1a_2}$ is a mismatch factor[42] which, from physical considerations, must lie between zero and unity.

In eq (70) the term M_{ga_1} of eq (69), has been written as M_{gA} since this is more in keeping with the new interpretation and notation. However, this reinterpretation does not change the load impedance which is presented to the generator, thus the two terms are identical. Comparison of eqs (69), (70), and (71) thus indicates that:

$$R_{a_1a_2} = \frac{N_{a_1a_2l} N_{a_2l}}{N_{Al}}. \tag{72}$$

An experimental determination of $R_{a_1a_2}$ may thus be effected by a measurement of the N's, as already described, and use of eq (72).

5. Relationship to Existing Art

The relationship between the new concept and the existing art can perhaps be best illustrated by an analogy. The usual circuit description of a microwave system (by means of the impedance or scattering parameters) represents a substantial simplification as compared with a description in terms of the field quantities. To a large degree this simplification is the result of suppressing a great deal of the information which is implicit in the field description, and focusing attention on those properties of the individual components which are observable at the respective terminal surfaces.

The new concept carries this simplification a step further by suppressing much of the detail which is involved in the circuit description, and instead substitutes the parameters P, η_a, M, N, R. These permit a system evaluation which is adequate for many practical purposes and because of their invariant properties the precision waveguide and connector requirement is avoided. It may be noted, in this context, that the usual specification for precision waveguide and connectors results not from the fundamental power transmission requirements, but rather to permit a system evaluation.

This elimination of the precision waveguide and connector requirement is significant both as a consequence of the elimination of this source of error and also in that it anticipates an application of existing technology to more general types of systems—e.g., those in which a well defined terminal plane between the different components may not exist.[43] In addition, while the described techniques for the measurement of P, η_a, M, N, R, are limited to the case where the perturbed cross section does not approach the point where multimode propagation becomes possible, the basic description (in terms of P . . . R) is applicable, and of potential value in multimode systems. In fact, the definitions of P_g,P_{gl}, M_{gl} are immediately applicable on a multimode basis. However because the *efficiency*, in a multimode problem, depends upon the generator as well as load parameters, the given definitions of η, N, R would need to be generalized.

In order to illustrate some of the advantages which are provided by the new concept, it is useful to consider the coaxial connector problem. For many years, the lack of a precision coaxial connector was a major limitation to accurate measurements in coaxial systems. The widely employed Type N connector suffered from the following problems: (1) impedance discontinuity, (2) lack of a well defined terminal surface, (3) its unipolar nature, (4) nominal differences between "identical" units due to manufacturing tolerances, and (5) lack of repeatability when the unit is disconnected and reconnected. Among this list of shortcomings, only the last is of concern in the new concept, where it is still necessary to postulate that a device can be removed and then replaced without changing the overall system.

Although the new concept calls for the substitution of mismatch factor measurement for impedance measurement, this does not imply that impedance measurements are obsolete. In particular the impedance concept makes it convenient to write system specifications in such a way that the mismatch factors will be nominally equal to unity and to identify the faulty unit if this design objective is not achieved. The mismatch factor measurement, on the other hand, permits a direct evaluation of how well the design objectives have been realized without regard for possible connector discontinuities.

Finally, the described techniques for measuring M, N, etc., have been developed on the assumption that the detectors on arms 3 and 4 of the g-reflectometer do not respond to phase information. In recent years, more sophisticated types of detection systems have been introduced for use in stepped frequency measurements, and which include phase sensitive detectors, memory, and a computer. Given this type of capability, a substantial simplification in the described procedures may be effected, but the development of this topic is outside the scope of this dissertation.

6. Summary

Historically, the quantities power, attenuation and impedance have been accepted as fundamental in the description and evaluation of microwave sys-

[42] Not to be confused with the radius of the circular locus introduced in eq (26).

[43] As previously noted, it is the (postulated) existence of a lossless region, where the interconnection takes place, that permits the terminal surface to remain arbitrary. Alternatively, the need for this lossless region may be avoided by specifying the location of the terminal surface.

tems. As noted in the preceding section, the usual precision waveguide and connector requirement is often imposed to permit the specification and evaluation in terms of these parameters, rather than as a result of more fundamental considerations.

The new concept, which has been outlined in this dissertation, proposes an alternative description in which power, maximum efficiency, and mismatch factor (M, N, R) are the fundamental parameters. The advantages of this concept are basically twofold. First, the inevitable departures from waveguide uniformity in any physically realizable system no longer impose a limitation on its experimental evaluation, and precision measurements are now possible in coaxial systems which are fitted with the Type N connector, for example. The second advantage of this concept is in the more easily understood physical model which it provides, where instead of taking account of the traveling wave amplitudes at the several ports, the attention is directed to the power transfer characteristics. (The terms M_{ga} and N_{al} in eq (19), for example, take the place of a rather complicated algebraic expression involving the six complex reflection and scattering coefficients.) As a consequence of this simplification, it should now be practical to instruct field personnel in the use and application of mismatch corrections.

It should perhaps be noted that many of the basic parameters, upon which the new concept is built, are not new. The quantities which are represented by P_g, η_a, M_{gl}, for example, are well known in the prior art. The contribution of this dissertation, is in the recognition of the invariant features of these parameters and the introduction of others such that independence of connector discontinuity is achieved. In addition, practical techniques, for the measurement of these terms, have been developed and described.

For completeness, it should be observed that while the analysis of nonuniform transmission lines and waveguides is a subject of current interest, the emphasis in this dissertation has been on the development of a measurement theory and practice rather than a detailed description of the fields associated with nonuniform waveguide. For convenience, the measurement procedures have been developed in terms of the existing circuit representation and then examined for their dependence upon uniformity requirements. In particular these procedures are characterized by their dependence only upon a phasable short, and methods for recognizing the equality of two "impedances." It is possible that these results could be obtained more directly from an alternative formulation.

Finally, the discussion has been limited primarily to the power measurement problem, with attenuation and impedance taking a supporting role. The potential application of this concept to phase measurement and multimode systems remains to be investigated.

7. Appendixes

7.1. Appendix 1

A formal demonstration of the invariance of M_{gl}, to a shift in terminal plane, may be effected with the help of figure 10. In particular it will be shown that M_{ga} (obtained at terminal 1) is equal to M_{al} (obtained at terminal 2) provided that the "attenuator" is lossless.

At terminal 1 in figure 10 the reflection coefficient, Γ_{in}, looking to the right is given by:

$$\Gamma_{in}=\frac{a\Gamma_l+b}{c\Gamma_l+1} \tag{A-1}$$

while at terminal 2, the reflection coefficient, Γ_g', is given by:

$$\Gamma_g'=\frac{a\Gamma_g-c}{-b\Gamma_g+1}. \tag{A-2}$$

The terms Γ_{in} and Γ_g' replace Γ_l and Γ_g respectively in eq (18) to yield M_{ga} and M_{al}.

For a lossless and reciprocal twoport, the parameters a, b, c, satisfy the conditions [44]

$$|a|=1 \tag{A-3}$$

$$b-ac^*=0. \tag{A-4}$$

Combining eqs (A–4, A–1), and (18) yields:

$$M_{ga}=1-\frac{|a\Gamma_l+ac^*-\Gamma_g^*-c\Gamma_l\Gamma_g^*|^2}{|1+c\Gamma_l-a\Gamma_l\Gamma_g-ac^*\Gamma_g|^2}. \tag{A-5}$$

Since $|a|=1$, the numerator of the second term may, without changing its value, be multiplied (inside the absolute value signs) by a*. Doing this, rearranging, and again using eq (A–4) yields,

$$M_{ga}=1-\frac{\left|\frac{a\Gamma_g-c}{-b\Gamma_g+1}-\Gamma_l^*\right|^2}{\left|1-\Gamma_l\left[\frac{a\Gamma_g-c}{-b\Gamma_g+1}\right]\right|^2} \tag{A-6}$$

which by comparison with eqs (A–2) and (18) is equal to M_{al}.

7.2. Appendix 2

Referring to figure 4, it will be shown that eq (11) is implied if the ratio P_3/P_4 remains constant for all positions of a sliding short on arm 2.

[44] This may be demonstrated in several ways. From physical considerations it is evident that $|\Gamma_{in}|=1$ when $|\Gamma_l|=1$. In terms of eqs (26) and (27) this calls for $R=1$, $R_c=0$, and which, with the appropriate changes in notation, leads to eqs (A–3) and (A–4).

With arm 2 terminated by a sliding short, b_2 and a_2 are related by

$$a_2 = b_2 e^{i\theta}. \tag{A-7}$$

Substitution in eq (15), and taking the absolute value yields,

$$\frac{P_3}{P_4} = \left|\frac{Ae^{i\theta}+B}{Ce^{i\theta}+D}\right|^2. \tag{A-8}$$

By hypothesis the ratio P_3/P_4 is a constant which will be denoted by K. Substitution of K and expansion of eq (A-8) yields,

$$|A|^2+|B|^2+AB^*e^{i\theta}+A^*Be^{-i\theta}$$

$$-K(|C|^2+|D|^2+CD^*e^{i\theta}+C^*De^{-i\theta})=0. \tag{A-9}$$

If this equation is to hold for arbitrary values of θ, the coefficients of the several powers of $e^{i\theta}$ must vanish independently. This leads to:

$$AB^*-KCD^*=0, \tag{A-10}$$

$$|A|^2+|B|^2-K(|C|^2+|D|^2)=0. \tag{A-11}$$

(The third equation is the complex conjugate of eq (A-10) and contains no additional information.)

Elimination of K between eqs (A-10) and (A-11) yields:

$$CD^*(|A|^2+|B|^2)=AB^*(|C|^2+|D|^2). \tag{A-12}$$

Multiplication of this expression by B/B^* and using the result $A^*BCD^*=AB^*C^*D$ (which follows from eq (A-10) since K is real) leads to:

$$A^2C^*D-AB(|C|^2+|D|^2)+B^2CD^*=0 \tag{A-13}$$

which may be factored to yield:

$$(AD-BC)(AC^*-BD^*)=0. \tag{A-14}$$

This equation admits of two solutions. However, the $AD-BC=0$ solution is trivial in the present context since it yields a value for P_3/P_4 which is completely independent of the terminating load. (As a practical matter, this adjustment would result if one of the couplers were reversed, and is useful in a different application.) This leaves the condition $AC^*-BD^*=0$, which was to be proved.

7.3. Appendix 3

It is the purpose of this appendix to demonstrate that for the usual reflectometer, the uniform waveguide requirement applies only to the output arm.

In figure 15 let there be an *enforced* current $a_1\mathbf{J}$ in one portion of the structure where $\mathbf{J}$ is a three dimensional vector function giving the spatial distribution and a_1 is a scalar multiplier. The current $a_1\mathbf{J}$ is the source of an electromagnetic field throughout the structure and is, by hypothesis, independent of the termination on the uniform section.

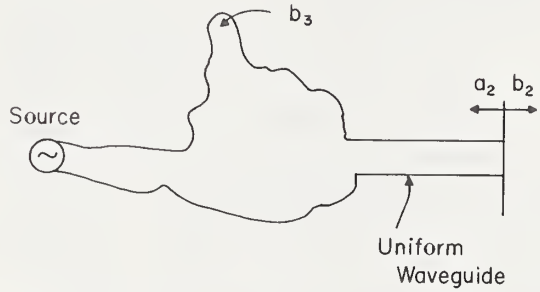

FIGURE 15. *Electromagnetic system used in deriving basic reflectometer equation.*

Let the uniform section (arm 2) be first terminated by a matched load. Then the field in arm 2 is completely specified by the term b_2 which is a measure of the electric field amplitude associated with the outgoing traveling wave. In another part of the structure, let the amplitude of the electric field component in an arbitrarily chosen direction be denoted by b_3. If the system is linear, it is possible to write,

$$b_2 = ha_1 \tag{A-15}$$

$$b_3 = ka_1 \tag{A-16}$$

where h and k are constants.

Next let an incoming wave, the amplitude of which will be denoted a_2, be launched in arm 2.[45] By superposition, b_2 and b_3 now become:

$$b_2 = ha_1 + ma_2 \tag{A-17}$$

$$b_3 = ka_1 + na_2 \tag{A-18}$$

where m and n are two additional constants. Elimination of a_1 between eqs (A-17) and (A-18) yields:

$$b_3 = Aa_2 + Bb_2 \tag{A-19}$$

where

$$A = \frac{hn-km}{h}, \tag{A-20}$$

$$B = \frac{k}{h}. \tag{A-21}$$

If, in the same way, the component of the field amplitude in a given direction, at another position, is denoted by b_4, it is possible to write,

$$b_4 = Ca_2 + Db_2 \tag{A-22}$$

[45] it makes no difference whether this is achieved by means of a source on arm 2, or by a partial reflection of the wave b_2.

where C and D are two additional constants of the system. Taking the ratio of eqs (A–19) and (A–22) yields eq (15), which is the starting point in the usual reflectometer development [3].

7.4. Appendix 4

It will be shown that the adjustment of T_x, to achieve the $C+D\Gamma_g=0$ condition, is invariant to the adjustment of T_y.

This could be done in a completely formal manner by finding the appropriate expressions for C and D in terms of the parameters of the two couplers and tuning transformers; this approach, however, is tedious. Instead, in figure 6, let an auxiliary terminal plane be temporarily inserted between the tuner T_y and the second coupler (the one on the right). Within this second coupler (including T_x) the electromagnetic fields are completely determined by a_2 and b_2 and, in particular, the field in arm 4 can be written

$$b_4=C'a_2+D'b_2 \tag{A–23}$$

where C' and D' are functions of the parameters of the second coupler and T_x (but not of T_y). If b_4 is eliminated between eqs (15) and (A–23) it becomes apparent that a nontrivial linear relationship exists between b_3, b_2, and a_2 if, and only if, $C/D=C'/D'$. Since the ratio C'/D' is invariant to the adjustment of T_y, the same is true of C/D. Thus, if this ratio is such that eq (14) is satisfied for one particular adjustment of T_y, the same is true for all possible adjustments.

7.5. Appendix 5

It is the purpose of this appendix to investigate, analytically, the wave propagation in a perturbed waveguide.

Beginning with figure 16 it will prove useful to consider the perturbed waveguide and a circumscribed uniform waveguide. Within the volume V, which is bounded by the uniform waveguide and the transverse surfaces S_1 and S_2, the electromagnetic field vectors, $\mathbf{E}$, $\mathbf{H}$, of course, satisfy Maxwell's equations which, for harmonic time dependence, may be written:

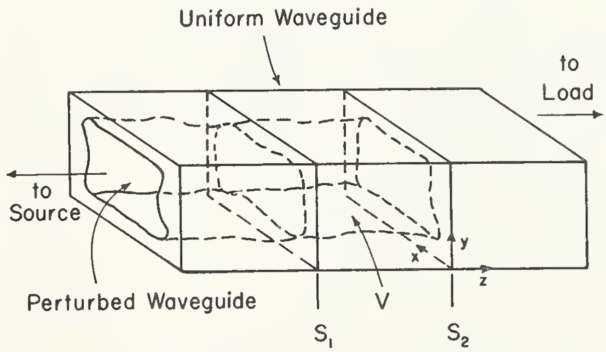

FIGURE 16. *System used to analyze wave propagation in perturbed waveguide.*

$$\nabla\times\mathbf{E}=-j\omega\mu\mathbf{H} \tag{A–24}$$

$$\nabla\times\mathbf{H}=j\omega\epsilon\mathbf{E}+\mathbf{J}, \tag{A–25}$$

where ϵ and μ are constant. In particular, the effect of the perturbed boundary is accounted for by the current density, $\mathbf{J}$. Taking the curl of eq (A–24) and substituting from eq (A–25) leads to

$$\nabla\times\nabla\times\mathbf{E}=k^2\mathbf{E}-j\omega\mu\mathbf{J} \tag{A–26}$$

where $k^2=\omega^2\mu\epsilon$.

In general the electric field $\mathbf{E}$ which satisfies eqs (A–24), (A–25) may be expanded in the form: [46]

$$\mathbf{E}=\sum_a\left[A_a(\mathbf{e}_a^0+\mathbf{E}_{az})e^{-\gamma_a z}+B_a(\mathbf{e}_a^0-\mathbf{E}_{az})e^{\gamma_a z}\right] \tag{A–27}$$

where a is a mode index, $\mathbf{e}_a^0$ and $\mathbf{E}_{az}$ are vector functions of the transverse (but not axial) coordinates, with $\mathbf{e}_a^0$ in the transverse plane and $\mathbf{E}_{az}$ in the axial direction, here assumed to be the z axis. In eq (A–27) the vectors

$$\overset{+}{\mathbf{E}}_a=(\mathbf{e}_a^0+\mathbf{E}_{az})e^{-\gamma_a z} \text{ and } \overset{-}{\mathbf{E}}_a=(\mathbf{e}_a^0-\mathbf{E}_{az})e^{\gamma_a z}$$

are the modal solutions of Maxwell's equations for the *unperturbed* waveguide. Finally, the complex scalar factors A_a and B_a are functions of z, but not of the transverse coordinates. Inspection of eq (A–27) shows that A_a and B_a represent the amplitudes of waves in the positive and negative z directions respectively.

In complete analogy with eq (A–27), the associated magnetic field $\mathbf{H}$ is given by:

$$\mathbf{H}=\sum_a\left[A_a(\mathbf{h}_a^0+\mathbf{H}_{az})e^{-\gamma_a z}-B_a(\mathbf{h}_a^0-\mathbf{H}_{az})e^{\gamma_a z}\right]. \tag{A–28}$$

Although a complete definition of the basis fields $\mathbf{e}_a^0$, $\mathbf{h}_a^0$ requires two normalization conditions, the discussion to follow requires only the power normalization which is assigned as follows:

$$\int_s \mathbf{e}_a^0\times\mathbf{h}_a^0\cdot\mathbf{n}\,ds=\frac{1}{2} \tag{A–29}$$

where the integration is over the transverse plane and $\mathbf{n}$ is in the positive z direction.

It is next necessary to introduce the vector analog of Green's second identity [47] which states that given a closed region of space, bounded by a regular sur-

[46] cf reference 16, pages 170–223.

[47] See for example, reference 15, p. 250.

face S, and two vector functions of position $\mathbf{P}$, and $\mathbf{Q}$, with continuous first and second derivatives, the following relationship holds:

$$\int_s (\mathbf{P}\times\nabla\times\mathbf{Q}-\mathbf{Q}\times\nabla\times\mathbf{P})\cdot\mathbf{n}\,ds \qquad \text{(A-30)}$$
$$\int_v (\mathbf{Q}\cdot\nabla\times\nabla\times\mathbf{P}-\mathbf{P}\cdot\nabla\times\nabla\times\mathbf{Q})dv$$

where $\mathbf{n}$ is now the outward normal from the surface which bounds the volume V

In eq (A-30), let the volume of interest be that which is bounded by S_1, S_2, and the unperturbed guide in figure 16. Then let:

$$\mathbf{P}=\mathbf{E}$$
$$\mathbf{Q}=\mathbf{E}_a^-=(\mathbf{e}_a^0-\mathbf{E}_{az})e^{\gamma_a z} \qquad \text{(A-31)}$$

which, with the help of eq (A-26), leads to

$$\int_s (\mathbf{E}\times\nabla\times\mathbf{E}_a^- -\mathbf{E}_a^-\times\nabla\times\mathbf{E})\cdot\mathbf{n}\,ds$$
$$=\int_v [\mathbf{E}_a^-\cdot(k^2\mathbf{E}-j\omega\mu\mathbf{J})-\mathbf{E}\quad\nabla\times\nabla\times\mathbf{E}_a^-]\,dv. \qquad \text{(A-32)}$$

As already postulated, $\mathbf{E}_a^-$ is a solution of Maxwell's equations, and eq (A-32) may be simplified with the help of the following:

$$\nabla\times\mathbf{E}_a^-=-j\omega\mu\mathbf{H}_a^- \qquad \text{(A-33)}$$

$$\nabla\times\nabla\times\mathbf{E}_a^-=k^2\mathbf{E}_a^- \qquad \text{(A-34)}$$

where

$$\mathbf{H}_a^-=(-\mathbf{h}_a^0+\mathbf{H}_{az})e\gamma a^z. \qquad \text{(A-35)}$$

After making these substitutions, eq (A-32) becomes:

$$\int_s (\mathbf{E}\times\mathbf{H}_a^- -\mathbf{E}_a^-\times\mathbf{H})\cdot\mathbf{n}\,ds=\int_v \mathbf{E}_a^-\cdot\mathbf{J}\,dv. \qquad \text{(A-36)}$$

Since the boundary conditions on a conducting surface require that the tangential electric fields vanish, the evaluation of the left side of this equation requires only the tangential components of $\mathbf{E}$ and $\mathbf{H}$ on the surfaces S_1 and S_2. If the tangential components on S_1 are represented by $\mathbf{E}_{1t}$ and $\mathbf{H}_{1t}$ respectively eqs (A-27) and (A-28) yield:

$$\mathbf{E}_{1t}=\sum_b [(A_{1b}e^{-\gamma_b z_1}+B_{1b}e^{\gamma_b z_1})\mathbf{e}_b^0] \qquad \text{(A-37)}$$

$$\mathbf{H}_{1t}=\sum_b [(A_{1b}e^{-\gamma_b z_1}-B_{1b}e^{\gamma_b z_1})\mathbf{h}_b^0] \qquad \text{(A-38)}$$

where z_1 is the value of z at terminal surface 1, and A_{1b}, B_{1b} are the coefficients of the expansion which give the required field at this terminal surface. If in eqs (A-37), (A-38) the subscript 1 is replaced by 2, the corresponding expression for the fields at terminal 2 is obtained.

Equation (A-36) may be now written

$$\int_{s1}\left\{\left[\sum_b (A_{1b}e^{-\gamma_b z_1}+B_{1b}e^{\gamma_b z_1})\mathbf{e}_b^0\right]\times(-\mathbf{h}_a^0 e^{\gamma_a z_1})\right.$$
$$\left.-e^{\gamma_a z_1}\mathbf{e}_a^0\times\left[\sum_b (A_{1b}e^{-\gamma_b z_1}-B_{1b}e^{\gamma_b z_1})\mathbf{h}_b^0\right]\right\}\cdot\mathbf{n}\,ds_1$$
$$+\int_{s2}\left\{\left[\sum_b (A_{2b}e^{-\gamma_b z_2}+B_{2b}e^{\gamma_b z_2})\mathbf{e}_b^0\right]\times(-\mathbf{h}_a^0 e^{\gamma_a z_2})\right.$$
$$\left.-e^{\gamma_a z_2}\mathbf{e}_a^0\times\left[\sum_b (A_{2b}e^{-\gamma_b z_2}-B_{2b}e^{\gamma_b z_2})\mathbf{h}_b^0\right]\right\}\cdot\mathbf{n}\,ds_2$$
$$=\int_v [\mathbf{e}_a^0-\mathbf{E}_{az}]e^{\gamma_a z}\cdot\mathbf{J}\,dv \qquad \text{(A-39)}$$

This expression may be simplified by cancellation and use of the orthogonality[48] relations:

$$\int_s \mathbf{e}_a^0\times\mathbf{h}_b^0\cdot\mathbf{n}\,ds=0. \qquad \text{(A-40)}$$
$$a\neq b$$

So that eq (A-39) becomes:

$$\int_{s1}2A_{1a}(-\mathbf{e}_a^0\times\mathbf{h}_a^0)\cdot\mathbf{n}\,ds+\int_{s2}2A_{2a}(-\mathbf{e}_a^0\times\mathbf{h}_a^0)\cdot\mathbf{n}\,ds$$
$$=\int_v(\mathbf{e}_a^0-\mathbf{E}_{az})e^{\gamma_a z}\cdot\mathbf{J}\,dv. \qquad \text{(A-41)}$$

Finally, observing that the outward normal from the volume V in figure 16 coincides with the positive z direction at S_2, but is opposite to it at S_1, and using eq (A-29) gives:

$$A_{2a}=A_{1a}-\int_v(\mathbf{e}_a^0-\mathbf{E}_{az})\cdot\mathbf{J}\,e^{\gamma_a z}\,dv. \qquad \text{(A-42)}$$

Equation (A-42) is interpreted as follows. At terminal 2 the coefficients A_{2a}, A_{2b}, A_{2c}, . . . represent a collection of waves whose direction of propagation is in the positive z direction. From eq (A-42) each of these coefficients is comprised of two parts . . . the complex amplitude which would obtain in the absence of the perturbation, plus a contribution due to the perturbation. Note that whereas the coefficients of the field expansion at terminals 1 and 2 are equal in the absence of the perturbation, the expression for the transverse field (cf. eq (A-37)) also includes the propagation constant $e^{\gamma_a z}$ such that the usual attenuation is obtained for those components of the field which represent evanescent waves.

[48] See reference 9.

With regard to the second term on the right of eq (A–42) it is convenient to choose the origin of the coordinate system such that $z=0$ on S_2. Then z is negative throughout the volume of integration. If in eq (A–42), A_{2a} represents the amplitude of one of the nonpropagating modes, the exponential factor indicates that the contribution to A_{2a} from the induced current density, $\mathbf{J}$, is small for those current elements which are remote from S_2. Indeed, it is both possible and convenient to visualize the induced current $\mathbf{J}$ as a secondary source of waves whose propagation is then governed by the dimensions of the unperturbed guide.

In the foregoing treatment, the only type of perturbation explicitly considered has been that of a deformed boundary, and this is certainly the problem of greatest practical interest. It is possible to obtain the same general conclusion if an arbitrary distribution of dielectric and magnetic media is also included in the perturbed guide.

This discussion provides the basis for the description given in section 3c.

7.6. Appendix 6

The derivation of eq (38) begins with the basic definition of N_{al} and help of eqs (21) and (23):

$$N_{al}=\frac{\eta_{al}}{\eta_a}=\frac{|S_{21}|^2(1-|\Gamma_l|^2)}{p-q|\Gamma_l|^2+z\Gamma_l+z^*\Gamma_l^*}\cdot\frac{p-q|\Gamma_m|^2+z\Gamma_m+z^*\Gamma_m^*}{|S_{21}|^2(1-|\Gamma_m|^2)} \tag{A–43}$$

$$=\frac{\left(1-\frac{q|\Gamma_m|^2}{p}+\frac{z\Gamma_m}{p}+\frac{z^*\Gamma_m^*}{p}\right)(1-|\Gamma_l|^2)}{\left(1-\frac{q|\Gamma_l|^2}{p}+\frac{z\Gamma_l}{p}+\frac{z^*\Gamma_l^*}{p}\right)(1-|\Gamma_m|^2)} \tag{A–44}$$

where:

$$p=1-|b|^2 \tag{A–45}$$

$$q=|a|^2-|c|^2 \tag{A–46}$$

$$z=c-ab^* \tag{A–47}$$

Before continuing with the development of the desired expression for N_{al}, it is necessary to obtain expressions for q/p and z/p as functions of Γ_m and η_r.

In terms of eqs (A–45), (A–46), (A–47), eq (35) becomes

$$z\Gamma_m|\Gamma_m|^2+(p-q)|\Gamma_m|^2+z^*\Gamma_m^*=0. \tag{A–48}$$

If this equation is subtracted from its complex conjugate, and noting that p and q are real, the resulting relation can be put in the form:

$$(z\Gamma_m-z^*\Gamma_m^*)(1-|\Gamma_m|^2)=0 \tag{A–49}$$

and since $|\Gamma_m|\leq 1$ for a passive termination, this requires, in general,[49]

$$z\Gamma_m=z^*\Gamma_m^*. \tag{A–50}$$

Equation (A–48) may thus be written:

$$\frac{z}{p}\Gamma_m(1+|\Gamma_m|^2)+\left(1-\frac{q}{p}\right)|\Gamma_m|^2=0. \tag{A–51}$$

Starting with eq (24), expanding, and using eqs (A–45), (A–46), (A–47) yields:

$$\eta_r^2=\frac{|a-bc|^2(1-|\Gamma_m|^2)^2}{(|1+c\Gamma_m|^2-|a\Gamma_m+b|^2)^2} \tag{A–52}$$

$$=\frac{(|a|^2+|bc|^2-|c|^2-|ab|^2+|c-ab^*|^2)(1-|\Gamma_m|^2)}{(p-q|\Gamma_m|^2+z\Gamma_m+z^*\Gamma_m^*)^2}$$

$$\eta_r^2=\frac{(pq+|z|^2)(1-|\Gamma_m|^2)^2}{(p-q|\Gamma_m|^2+z\Gamma_m+z^*\Gamma_m^*)^2} \tag{A–53}$$

Equation (A–48) may be rearranged in two different ways to yield:

$$q|\Gamma_m|^2-z^*\Gamma_m^*=(p+z\Gamma_m)|\Gamma_m|^2 \tag{A–54}$$

and

$$|z|^2=(q-p)z\Gamma_m-z^2\Gamma_m^2. \tag{A–55}$$

Substitution of these expressions in the denominator and numerator respectively of eq (A–53) leads to:

$$\eta_r^2=\frac{\frac{q}{p}-\frac{z}{p}\Gamma_m}{1+\frac{z}{p}\Gamma_m}. \tag{A–56}$$

Equations (A–51) and (A–56) may now be solved simultaneously to yield the following expressions for z/p and q/p,

$$\frac{z}{p}=-\frac{(1-\eta_r^2)\Gamma_m^*}{1-\eta_r^2|\Gamma_m|^2} \tag{A–57}$$

$$\frac{q}{p}=\frac{\eta_r^2-|\Gamma_m|^2}{1-\eta_r^2|\Gamma_m|^2}. \tag{A–58}$$

Finally, eqs (A–57) and (A–58) may be substituted in eq (A–44) which may then be factored to obtain eq (38).

Equation (39) may be obtained from eq (38) with the help of the relationship:

$$(1-|\Gamma_l|^2)(1-|\Gamma_m|^2)=|1-\Gamma_l\Gamma_m^*|^2-|\Gamma_l-\Gamma_m|^2 \tag{A–59}$$

which may be confirmed by the expansion of both sides.

[49] This same result may be obtained from eq (A–43) where the second factor on the right is the reciprocal of η_a. By inspection η_a is a maximum with respect to the argument of Γ_m when $z\Gamma_m$ is real and negative.

7.7. Appendix 7

Equation (42) may be obtained as follows. Starting with eqs (27) and (29), R_{c1} is given by:

$$R_{c1}=\frac{|(B-Ac)(D-Cc)^*-(Aa-Bb)(Ca-Db)^*|}{||D-Cc|^2-|Ca-Db|^2|}. \tag{A-60}$$

Combining this with eq (31), and imposing the condition $BD^*-AC^*=0$, yields:

$$\frac{R_{c1}}{R_2}=\frac{\left|\frac{z}{p}\Gamma_l^2+\left(1-\frac{q}{p}\right)\Gamma_l+\frac{z^*}{p}\right|}{\left|1-\frac{q|\Gamma_l|^2}{p}+\frac{z\Gamma_l}{p}+\frac{z^*\Gamma_l^*}{p}\right|} \tag{A-61}$$

where p, q, and z are defined by eqs (A–45), (A–46), (A–47).

Substituting the values of z/p and q/p from eqs (A–57) and (A–58) into this expression gives eq (42).

7.8. Appendix 8

It is the purpose of this appendix to determine the conditions under which a reflection coefficient of magnitude less than unity will be mapped into the interior of the circle of figure 13.

Returning to eq (27), it will prove convenient for this appendix to interpret R_c as a *complex* number, which locates the center of the transformed circle in the w plane. Then:

$$R_c=\frac{\beta\delta^*-\alpha\gamma^*}{|\delta|^2-|\gamma|^2}. \tag{A-62}$$

By inspection of eq (25), the origin of the z plane transforms to the point β/δ in the w plane. Provided that this point is within the transformed circle, continuity considerations insure that all values of z, for which $|z|<1$ will be mapped to its interior. Evidently, the point β/δ will be found within the transformed circle provided that:

$$\left|R_c-\frac{\beta}{\delta}\right|<R. \tag{A-63}$$

Substituting from eqs (26) and (A–62) gives

$$\left|\frac{\beta\delta^*-\alpha\gamma^*}{|\delta|^2-|\gamma|^2}-\frac{\beta}{\delta}\right|<\frac{|\beta\gamma-\alpha\delta|}{||\delta|^2-|\gamma|^2|} \tag{A-64}$$

which after simplification becomes

$$|\gamma|<|\delta|. \tag{A-65}$$

The application of this result to the g-reflectometer yields

$$|C|<|D|. \tag{A-66}$$

As a practical matter, this relationship is well satisfied by the type of system described, and in any case must hold if the tuning condition called out in eq (14) is realized.

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